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THE

PHYSICAL REVIEW.

THE TEMPERATURE COEFFICIENTS OF GAS VISCOSITY.

By Willard J. Fisher.

I. Sutherland's Equation, Pure Gases.

THE researches of Graham[1] were among the first from which it could be with certainty deduced that gases do not flow as perfect fluids, but have a definite internal friction. This was explained on the basis of the Kinetic Theory by J. Clerk-Maxwell,[2] who showed that it was to be anticipated that the coefficient of viscosity for a given temperature, would be independent of the pressure, and for a given pressure, would vary as the square root of the absolute temperature. The first of these deductions was verified by Clerk-Maxwell himself,[3] O. E. Meyer,[4] and others,[5] using the vibration method, which is best adapted to this purpose; and the limits within which it is true were indicated by Crookes,[6] and Kundt and Warburg, who showed that the statement fails for very low pressures. Warburg and Babo[7] showed the same for CO_2 at high pressures.

The second deduction was tested by O. E. Meyer,[8] Stefan,[9] E. Wiedemann,[10] Puluj,[11] von Obermayer,[12] S. W. Holman,[13] and C. Barus,[14] and decided variations from it were found to be true. In fact, Holman showed that for air and CO_2, the variation of viscosity was not proportional to any numerical power of the absolute temperature.

At this point W. Sutherland[15] showed that "the whole discrep-

ancy between theory and experiment disappears if account is taken of molecular force, which brings about collisions which would have been avoided in its absence," and justified his conclusions by examining the data of preceding experimental investigations. His theory led to a relation of the form.

$$\eta \propto \frac{\theta^{\frac{1}{2}}}{1 + \frac{C}{\theta}}$$

in which η is the viscosity-coefficient, θ the absolute temperature, and C a constant, which is the potential energy of two molecules of the gas at their position of nearest approach, "in contact"; C has the dimensions of temperature; the formula is given for conditions such that the temperatures are above the critical and the pressures within the range of approximate exactness of Boyle's Law.

The latter reaches of Rayleigh[16], Breitenbach[17], Schultze[18], Markowski[19], Bestelmeyer[20], Kleint[21], and Tänzler[22], have further justified Sutherland's theory for the ranges of temperature and pressure for which it was proposed.

Taking η_0, the viscosity-coefficient at 0° C., as a second constant, all writers hitherto have thrown Sutherland's relation into the form

$$\eta = \eta_0 \left(\frac{\theta}{273}\right)^{\frac{1}{2}} \frac{1 + \frac{C}{27}}{1 + \frac{C}{\theta}}$$

and determined η_0 and C by the method of least squares. It can, however be equally well written with a proportionality-constant in the form

$$\eta = \frac{K\theta^{\frac{1}{2}}}{1 + \frac{C}{\theta}}.$$

The constant K has not hitherto received any notice, no one having even determined its numerical value for any gas; but both it and C have interesting properties. Its dimensions are $[M]\,[L]\,[T]^{-1}\,[\theta]^{-\frac{1}{2}}$; and if the dimensions of θ are the same as those of kinetic energy, then $[K] = [M]^{\frac{1}{2}}$.

Sutherland's equation may be thrown into the form

$$\theta = K\frac{\theta^{\frac{1}{2}}}{\eta} - C,$$

which is linear in terms of θ and $\theta^{\frac{1}{2}}/\eta$, with the slope equal to K, and the ordinate-intercept equal to $-C$. This suggests the plotting of the fundamental data, and the determination of the constants from the plots. For such of the investigations as have already been reduced by least squares the value of K is perhaps better deduced from the published values of η_0 and C; for some which have not, the writer has used the graphical method, finding by trial that it gives a very good agreement with the other. But the curves in themselves are interesting, and are shown, on small scale, for elements in Fig. 1, for compounds in Fig. 5. Table I. gives the fundamental data for the figures, and covers all the gases which have been studied through a sufficiently wide range above their critical temperatures. Table II. gives a summary of certain properties of these gases, arranged in the order of their molecular weights.

TABLE I.

FUNDAMENTAL DATA ON GAS VISCOSITY.

von Obermayer,[11] *note 1.*

	Temp. C. (Means.)	$\eta \cdot 10^7$ (Means.)	θ	$\frac{\theta^{\frac{1}{2}}}{\eta} \cdot 10^{-7}$
Carbon Monoxide, CO.	17.7	1703	290.7	2.905
	53.45	1857	326.45	3.175
Nitrous Oxide, N_2O.	−21.5	1249	251.5	3.192
	14.05	1421	287.05	3.423
	53.55	1606	326.55	3.675
	100.3	1829	373.3	3.944
Ethylene, C_2H_4.	−21.5	851	251.5	4.686
	20.6	989	293.6	5.085
	53.5	1096	326.5	5.381
Carbon Dioxide, CO_2.	−21.5	1278	251.5	3.120
	14.25	1449	287.25	3.354
	53.5	1638	326.5	3.601
	102.3	1858	375.3	3.914
	162.4	2143	435.4	4.239
	222.0	2385	495.0	4.617
	240.3	2458	513.3	4.732

P. Breitenbach,[16] *note 2.*

Hydrogen.	−20.6	819	252.4	4.907
	15.0	889	288.0	5.498
	99.2	1059	372.2	6.781
	182.4	1215	455.4	8.000
	302.0	1392	575.0	9.907
Ethylene, C_2H_4.	−21.2	891	251.8	4.49
	15.0	1006	288.0	4.86
	99.3	1278	372.3	5.625
	182.4	1530	455.4	6.355
	302.0	1826	575.0	7.56
Carbon Dioxide, CO_2.	−20.7	1294	252.3	3.100
	15.0	1457	288.0	3.355
	99.1	1861	372.1	3.861
	182.4	2221	455.4	4.380
	302.0	2682	575.0	5.140
Methyl Chloride, CH_3Cl.	−15.3	936	257.7	4.415
	15.0	1052	288.0	4.64
	99.1	1384	372.1	5.185
	182.4	1706	455.4	5.70
	302.0	2139	575.0	6.455

H. Schultze,[17] *note 3.*

Argon. Series I.—2⅓ % Nitrogen.	17.9	2241	290.9	2.215
	99.8	2740	372.8	2.645
	183.7	3178	456.7	3.075
Series II.—½ % Nitrogen.	14.7	2208	287.7	2.208
	99.7	2733	372.7	2.638
	183.7	3224	456.7	3.025
Helium + 1⅔ % Ne (Vol.).	15.3	1969	288.3	2.487
	99.6	2348	372.6	3.062
	184.6	2699	457.6	3.627

H. Markowski,[18] *note 4.*

Oxygen.	16.75	2023	289.75	2.438
	99.74	2485	372.74	2.896
	185.8	2885	458.8	3.406
Hydrogen.	14.5	877	287.5	5.558
	100.5	1046	373.5	6.901
	184.2	1212	457.2	8.066
Nitrogen, chemically prepared.	15.4	1747	288.4	2.806
	100.08	2123	373.08	3.388
	182.7	2458	455.7	3.956

E. Wiedemann,[9] *note 5.*

	8°	100°	184.5°
Air	100	123.1	141.1
CO	96.87	—	136
CO_2	80.48	104.8	123.4
N_2O	80.41	105.6	124.1
C_2H_4	56.24	73.89	87.38

S. W. Holman,[12] *Note 6.*

Carbon Dioxide, Series II.

Temp. C.	18.10	40.98	59.14	79.65	100.17	119.30
η/η_0	1.0679	1.1458	1.2126	1.2848	1.3506	1.4148
Temp. C.	141.89	158.28	181.32	224.0	17.07	
η/η_0	1.4843	1.5371	1.6188	1.7474	1.0630	

Empirical Formula for the above:

$$\eta/\eta_0 = 1 + 0.003725t - t^2 \cdot 264 \cdot 10^{-8} + t^3 \cdot 417 \cdot 10^{-11}.$$

A. Bestelmeyer,[19] *Note 7.*

Nitrogen, Chemically Prepared.

Temp. C.	300.4	98.41	−78.66	−190.63
η/η_{17}	1.6279	1.2064	0.7207	0.3204

NOTES TO TABLE I.

The values of $\theta^{\frac{3}{2}}/\eta$ were computed with a very good ten-inch slide-rule, which gives sufficiently accurate results for Fig. 1. In case swhere least square values were not available, the slide-rule values were corrected with four-place logarithms as noted below, to insure the accuracy of the large-scale plots from which K was determined. The observations given have been selected from the large mass available (of which most is presented in Landolt and Börnstein, 1905) with reference mainly to the range of temperature; this necessarily excludes the results obtained by vibrations, and most of the data of Graham[1] and the other earlier observers who used the transpiration method. In a few cases where the temperature range was small but the results unique the value of C was computed to see whether it was possible, and if not, the observations were not included. Observations like those of Graham on chlorine and S. Koch[23] on mercury vapor, and others in which the temperature range was not sufficiently elevated above the critical temperature, have also been rejected for this purpose.

Note 1, von Obermayer's Data.—The transpiration apparatus used has been criticized on account of insufficient accuracy in the calibration of the capillaries, and lack of certainty that the gas had assumed the temperature of the capillaries before entering the same. The temperatures were obtained by liquid and vapor baths, and measured by means of mercury thermometers. Constancy of pressures was secured by a special form of apparatus. Many runs at different temperatures and with different capillaries were made; the means for the table were computed from these by averaging the (nearly equal) temperatures and viscosity values. These values are not considered as good as those of Graham, or some of the investigators of later years. The data given for CO are nearly unique, and are here quoted for that reason, in spite of the fact that the range of temperature is too small to allow final conclusions on the values of the constants. The values of $\theta^{\frac{3}{2}}/\eta$ given have

been determined throughout from von Obermayer's results by logarithms, and plotted on cross-section paper to a scale of 1/20 inch to 1°C. for ordinates, 1/20 inch to 1/100 unit of $\theta\sqrt[3]{}/\eta \cdot 10^{-7}$ for abscissas. In drawing the curves temperatures below the critical were neglected. This leaves only two points for N_2O and C_2H_4, but gives a good series for CO_2.

The points for CO_2 do not lie very exactly on the straight line, but vary on the two sides somewhat. The two points for the temperatures —21°.5 and 14°.25 lie above the line, the lower more than the upper, indicating below the critical temperature a relative increase in the viscosity over that expressed by the formula. From the slope and intercept computed from two widely separated points on the line the constants are found to be $K = 165.8\ 10^{-7}$, $C = 271.5$.

Breitenbach's values plotted on the same sheet give a very accurate straight line, which intersects the line for von Obermayer's data at 182°.4 C., and has a less slope, hence also a less value of C. The comparison of the various determinations of the constants of CO_2 is continued below, Note 6.

For N_2O the constants computed are $K = 173.8 \cdot 10^{-7}$, $C = 312$.

For C_2H_4, $K = 111.1 \cdot 10^{-7}$, $C = 271.6$.

For CO, $K = 135.1 \cdot 10^{-7}$, $C = 102$.

The temperature ranges for these three gases are hardly sufficient to settle any of the values.

Note 2, Breitenbach's Data.—These important researches include, beside those on pure gases given in the table, others on mixtures of H with CO_2 and CH_4 with air, also on the slipping of gases on the walls of the capillary. The apparatus was after the model used by E. Wiedemann, and, was evidently, with additions to increase symmetry and convenient use, the pattern followed by H. Schultze, mentioned in the next note. No full descriptions of the methods of preparation of the gases are given, but reference is made to the dissertation of which the Annalen article is an abstract. The results agree very well with the Sutherland formula, so far as it is applicable. Following are Breitenbach's Least Square values for the constants:

Hydrogen, $\eta_0 = 857 \cdot 10^{-7}$, $C = 71.7$; from which $K = 65.5 \cdot 10^{-7}$.
Ethylene, $\eta_0 = 961 \cdot 10^{-7}$, $C = 225.9$; from which $K = 106.2 \cdot 10^{-7}$.
CH_3Cl, $\eta_0 = 989 \cdot 10^{-7}$, $C = 454$; from which $K = 159.4 \cdot 10^{-7}$.
CO_2, $\eta_0 = 1388 \cdot 10^{-7}$, $C = 239.7$; from which $K = 157.8 \cdot 10^{-7}$.

The results on CH_3Cl are especially interesting, as they include a temperature within ten degrees of the boiling-point of the substance, and cover the large range of 317°.

Note 3, H. Schultze's Data.—The apparatus used was of the form probably best adapted to absolute determinations of viscosity, being symmetrical about a single capillary, mercury being used to force the gas through in either direction; the pressures were adjusted by screws raising or lowering the mercury reservoirs, and read by compound mercury-water manometers; temperatures were kept constant by vapor-baths of steam or aniline; volumes were determined by calibration of the two gas-pipettes, and the time of flow was determined by the rising mercury automatically ringing an electric bell. The capillary was calibrated by a method, apparently original, which takes account of irregularities in the tube, and allows of but few improvements. The ratio of length to diameter of tube was 3469 6, which is above the minimum, 3000, set by O. E. Meyer for such work. In computing, account was taken of the different temperatures of the air in the different parts of the apparatus, and also of the coefficient of slip.

The second series for argon is no doubt the better, on account of the greater purity; from it the constants deduced are $\eta_0 = 2104 \cdot 10^{-7}$, $C = 169.9$; from which $K = 206.5 \cdot 10^{-7}$; $C/K = 8.23 \cdot 10^6$.

The helium used was made from monazite sand, and carefully purified from CO_2, H, N and hydrocarbons; the inert Ne remaining could not be removed for lack of liquid H. The results are to be accepted with some reservations, which are pointed out by the investigator himself; chief among these are the mixture with Ne, which may have upon He the same effect that heavier gases have upon H, small per cents. increasing the viscosity abnormally. The Ne certainly produced some effect of importance, as the observations made after the apparatus had stood for some time gave higher results than those in which one run followed immediately upon another. Schultze has observed the same with mixtures of N and H.

His constants for this impure helium are, $\eta_0 = 1891.10^{-7}$, $C = 80.3$; from which $K = 148.2 \cdot 10^{-7}$.

Rayleigh's values of C, 150.2 for argon and 72.2 for helium, differ considerably from the above. He had the advantage of purer gases, but his apparatus may be criticized, as by Schultze in his article on helium.

It should be noted that the portion of the work of P. Tänzler [22] which deals with pure helium leads him to results in exact agreement with these of Schultze, namely, $C = 80.3$, $\eta_0 \cdot 10^7 = 1891$. On pp. 224–5 he says: "The density determined gave

for argon 19.946 referred to O = 16,
for helium 2.066 referred to O = 16.

As spectrometric investigations in our Institute have shown that helium usually contains argon as an impurity (even 0.573 per cent. by volume), I have consequently in the calculation of the percentage content of mixtures of helium and argon assumed the density of pure helium to be 1.98." Hence it may be doubted that the uncertainty regarding the values for helium is entirely removed, as the gas which he calls "He 100 %" in his table may have contained sufficient impurity to give the same results as the mixture used by Schultze.

Note 4, Markowski's Data. — The apparatus was that constructed by H. Schultze, and so the results are in absolute measure. The gases were prepared with great care, but it would seem with more successful results in the cases of oxygen and nitrogen than in that of hydrogen. On the agreement of the oxygen series with that of von Obermayer, the latter is not probably the better. Rayleigh's value of C for oxygen, 128.2, was found by experiments with apparatus which is not above criticism both on theoretical and on practical grounds (see H. Schultze,[17] second article).

From his experiments Markowski deduces for oxygen the values $\eta_0 = 1926.10^{-7}$, $C = 138$. . From these the writer computes $K = 175.5 \cdot 10^{-7}$, $C/K = 7.865 \cdot 10^6$.

While the hydrogen used was electrolytically prepared, the results obtained do not agree with Sutherland's formula, as is shown graphically by the curve of Fig. 1, where the points are the vertices of a triangle and not in a straight line. The writer would conclude that the results of Breitenbach are to be preferred to these.

Markowski gives as the constants for hydrogen, $\eta_0 = 841 \cdot 10^{-7}$, $C = 83.0$, which yield $K = 66.4 \cdot 10^{-7}$.

On the remarkable agreement of the results for nitrogen with those of A. Bestelmeyer, see note 7. The least square constants are $\eta_0 = 1674 \cdot 10^{-7}$, $C = 113$; from which $K = 143.3 \cdot 10^{-7}$, $C/K = 7.89 \cdot 10^6$.

Note 5, E. Wiedemann's Data. — The transpiration apparatus used was apparently adapted to give relative results only; one of the two tables of data (both of which are derived from the same observations) is presented here. From it are computed the following values for C, due weight being allowed in the computation to the longer tem-

perature intervals, and temperatures below the critical not being used: CO, 87.0; C_2H_4, 242; N_2O, 219.5; CO_2, 177. These differ materially from the results obtainable from other investigations, as seen in Table II.

Note 6, S. W. Holman's Data. — This observer's results on air and CO_2 are derived from one of the longest and most careful investigations in the history of the subject, and may be considered standard. The transpiration apparatus with tandem capillaries was of the best form for the derivation of relative results, the pressure regulator of very ingenious design and great efficiency, the mercury thermometers carefully calibrated, and all the work of observation, computation and criticism carried through with a finish worthy of its author. He conclusively proved that constant powers of the absolute-temperature could not be used to express his results commensurately with their precision; and it was especially on the above quoted series for CO_2 that Sutherland based his first value of C. The method of computation then used does not appear to the present writer to be as good as the following.

Assuming Holman's empirical formula, and substituting in it 100° C., we obtain for the ratio η_{100}/η_0 the value 1.3503. Equating this to Sutherland's form for the same ratio, and solving for C, we have $C = 274.2$, as against Sutherland's value derived from the same data, 277. It is unfortunate that the series is insufficient for the determination of K.

Of the various determinations of the constants of CO_2, it is safe to discard at once that of E. Wiedemann, unless the original observations are to be recomputed. C from Holman's data, 274, agrees very well with 271.5 derived from von Obermayer's; but the plot of the latter shows a greater deviation series than that of Breitenbach, so that on the whole the Sutherland formula represents the latter better. Quite possibly the real values for CO_2, both of K and C, are to be regarded as still somewhat in doubt, and subject to further experimental investigation.

Note 7, Bestelmeyer's Data. — The transpiration apparatus used was designed after that of Holman, with tandem capillaries, thus rendering unnecessary the calibration of the tubes. Holman determined experimentally the ratio of the tube-constants; Bestelmeyer also calibrated his tubes by the method of H. Schultze, both before and after the experiments on viscosity. The pressure-regulating apparatus seems to have worked very well, and the temperatures were very accurately determined over the great range of 491° from that of boiling diphenylamine to that of liquid air,— below the critical temperature of nitrogen. This is the largest range of temperature ever used in viscosity work, and by its means a relative increase of viscosity below the critical temperature is discovered in the case of nitrogen as in other gases and vapors below their critical temperatures.

The table quoted is one of mean values, giving averages of the experimental data and comparing with them the values computed by least squares. From four sets of runs Bestelmeyer derives for C the values 110.6, 110.3, 114.4 and 103.6, the first three of which agree well among themselves and with Markowski's value, 113, obtained from absolute measurements with a very different apparatus. If we compute from Markowski's least square data the value of η at 15° C., and with this and Bestelmeyer's experimental values of η/η_{15} compute absolute values of η, the plotting to very large scale of these and Markowski's absolute values gives a remarkable coincidence, the two series forming one curve. Hence it may be concluded that the viscosity coefficient of nitrogen is now known more accurately and over a larger range of temperature than that of any other gas, and that further investigation of it will be unnecessary for a long time to come.

TABLE II.

Constants of Gases whose Viscosity has been Studied through a Wide Range of Temperature.

Formula.	H_2	A	He	A	N_2	A	O_2	A	Ar	A
Density, Air = 1, 760 mm. (*L*)	0.0693	Rt	0.1368	R&T	0.967	Ldc	1.105	Ldc	1.379	R&T
Molecular weight, *O* = 16. (*L*)	2.016		4		28.08		32		39.9	
Boiling point, 760 mm. (*L*)	−253	T	−267	T	− 195.7	F&A	− 182.8	T	−186.1	R&T
Critical temperature. (*L*)	−234.5	O	<−264	D	− 146	W	− 118.0	W	−117.4	R&T
	−243 to				− 146.5	W	− 118.8	O		
	−241	D			− 146	O				
$\eta \cdot 10^7$, 0°C.	861	v.O	1891	Sch	1659	v.O	1873	v.O	2104	Sch
(Quoted from Authors.)	857	B			1674	M	1926	M		
	841	M								
Viscosity constants. $K \cdot 10^7$.	65.5	B	148.2	Sch	143.3	M	175.5	M	206.5	Sch
	66.4	M								
C (Quoted from Authors).	79	S	72.2	R	113	M	127	S	169.9	Sch
	71.7	B	80.3	Sch	110.6	Br	138	M	150.2	R
	72.2	R			110.3	"	128.2	R		
	83.0	M			114.4	"				
					103.6	"				
Mean C.	76.5				110.4		131.1			
Extreme temperature range	−20.6 to		15.3 to		−190.6 to		15.4 to		14.7 to	
for η.	302		184.6		300.4		185.8		183.7	Sch

Formula.	CO	A	C_2H_4	A	CO_2	A	N_2O	A	CH_3Cl	A
Density, Air = 1,760 mm. (*L*)	0.967	Ldc	0.985	Ss	1.5291	R	1.527	Thd	1.731	D
Molecular weight, *O* = 16. (*L*)	28		28.03		44.0		44.08		50.47	
Boiling point, 760 mm. (*L*)	−190	O	−153	(B)	−78.2	Rt	−87.9	Rt	−23.7	(B)
Critical temperature. (*L*)	−141.1	W	9.2		30.92		35.4	D	141.5	V&C
	−139.5	O	to	*	to	*	36.4	Jn		
			13.0		31.9		38.8	Vd		
$\eta \cdot 10^7$, 0° C.	1625	v.O	922	v.O	1387	v.O	1353	v.O	989	B
(Quoted from Authors.)			961	B	1388	B				
			966	G						
Viscosity constants. $K \cdot 10^7$.	135.1	v.O	106.2	B	157.8	B	173.8	v.O	159.4	B
			111.15	v.O	165.8	v.O				
C (Quoted from Authors).	102	v.O	272	S	274	H	219.5	Wn	454	B
	87	Wn	225.9	B	239.7	B	312	v.O		
			242	Wn	177	Wn				
					271.5	v.O				
Extreme temperature range	17.5 to	v.O	21.2 to		20.7 to		−21.5		−15.3 to	
for η.	53.5		302	B	302	B	to 100.3	v.O	302	B

* Selected from many, too numerous to quote.

TABLE II. — AUTHORITIES.

B, Breitenbach.
(B), Biedermann's Chemiker Kalendar, 1906.
Br, Bestelmeyer.
D, Dewar.
F & A, Fischer and Alt.
G, Graham.
H, Holman.
Jn, Janssen.
Kn, Knietsch.
L, Lussana.
(L), Landolt und Börnstein, 1905.
Ldc, Leduc.
M, Markowski.
O, Olszewski.
R & T, Ramsay and Travers.
Rt, Regnault.
S, Sutherland.
Sch, Schultze.
Ss, Saussure.
T, Travers.
Thd, Thenard.
V & C, Vincent and Chappuis.
Vd, Villard.
v.O, von Obermayer.
W, Wroblewski.
Wn, E. Wiedemann.

Values of K computed by present writer, from data of authors indicated.

In plotting Figs. 1 and 5 the lines were first drawn from the values of K and C given in Table II., then the points representing observations were marked; so that the lines represent least square computations in almost all cases, von Obermayer's results alone having been reduced graphically. In Figs. 2, 3 and 4, the points were marked first from the same data and standard values of molecular weights, and the lines then drawn among the points in the positions deemed to be most suitable.

With regard to the relative value of least square as compared with large scale graphical reductions, it is no doubt true that the ordinary methods of least square work assume that one of the coördinates is measured without error, or else that the measurements of both coördinates are of equal weights. It is hardly to be supposed that either of these conditions is common; in the study of gas viscosity, for example, the number of sources of error affecting the temperature values is not nearly as large as that affecting the viscosity coefficient, and it is not to be supposed that values of θ and η are by any means of equal weight; while it is certain that θ can not be properly assumed without error. Unless some account is taken of these principles, the extra labor involved in a least square reduction is mostly thrown away; if authors use least squares, they should at least state what relative weights they assign their coördinates.

From the tables and these curves certain deductions may be drawn, though no final conclusions; and indications for profitable research work.

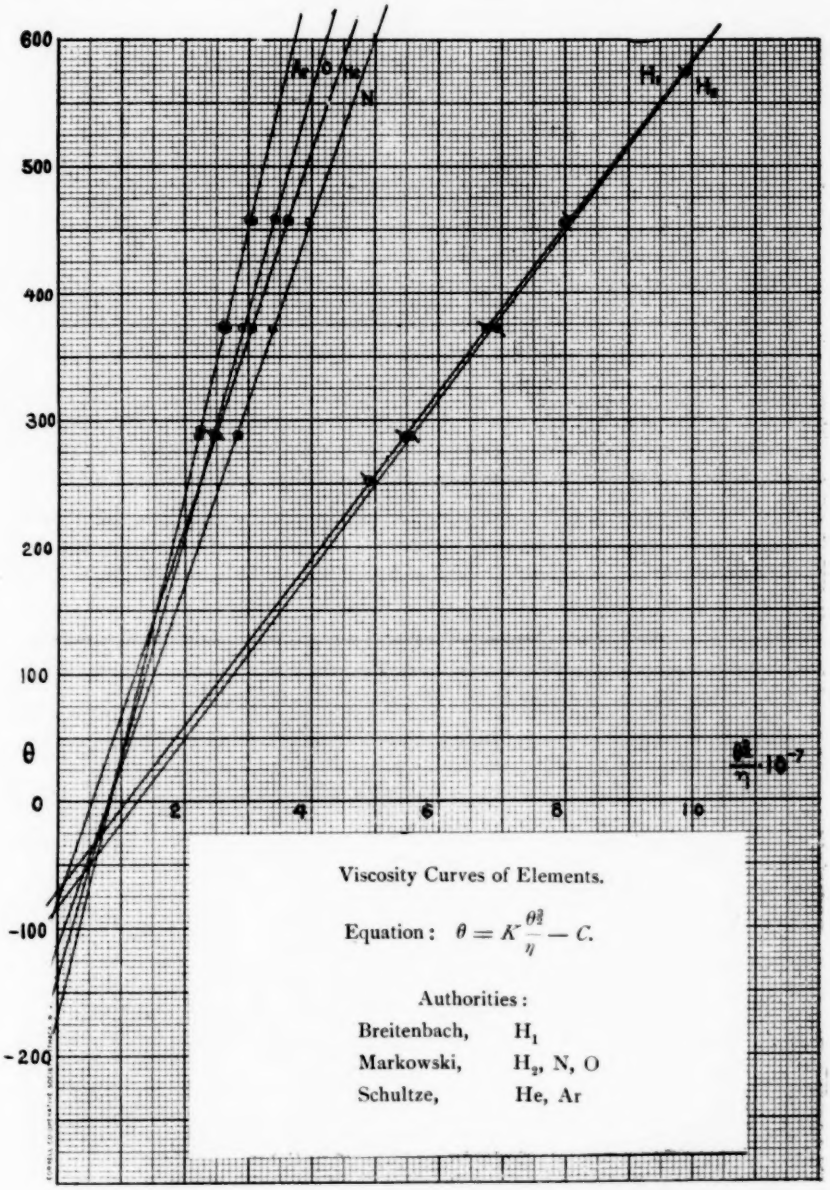

Fig. 1.

Fig. 1 shows that for the elements the curves have a general converging tendency, not toward the origin but a point in its neighbor-

hood, and that both the slope and the y-intercept increase with increasing molecular weight. The presence of a positive, not a zero, x-intercept, shows that the ratio θ^2/η approaches a definite limit as θ approaches zero, or would do so if the law did not change at or about the critical temperature. The convergency of the lines is best shown by N, O and Ar, which come together almost on the

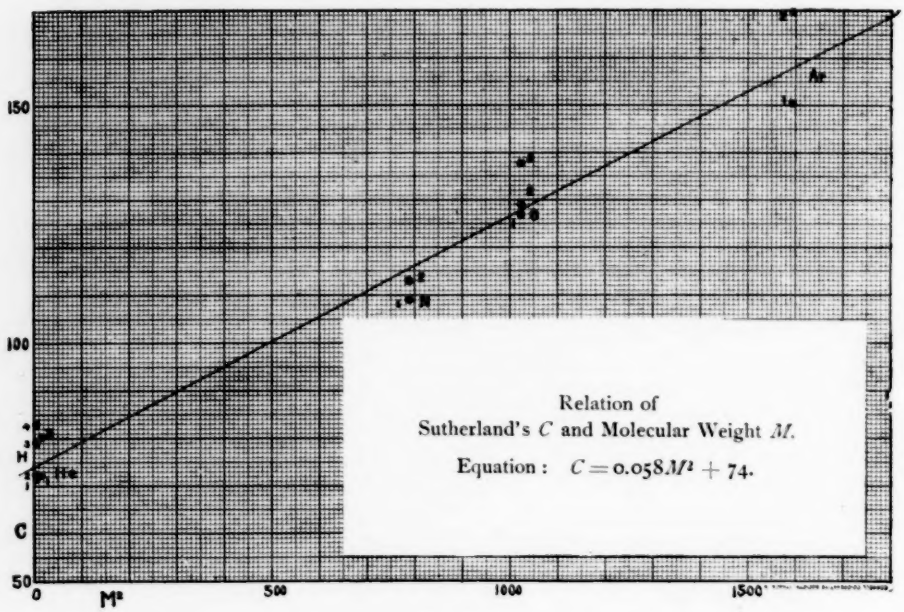

Fig. 2.

Authorities :

H. (1) Breitenbach, (2) Rayleigh, (3) Sutherland, (4) Markowski.
He. (1) Rayleigh, (2) Schultze.
N. (1) Bestelmeyer, (2) Markowski.
O. (1) Sutherland, (2) Rayleigh, (3) Markowski.
Ar. (1) Rayleigh, (2) Schultze.

x-axis, with a common abscissa nearly equal to $8 \cdot 10^6$. While H and He show the same convergent tendency, the discrepancy of the lines representing different investigations on H, and the differences of the numerical values in the tables, tend to show that its constants are not as yet well known; and further investigation might possibly show that the absolutely pure gas would give a plot crossing the axis nearer the point for N, O and Ar. No certain conclusion

can be drawn from the data or curve for He, as it was the result of work with impure gas; though it would not be surprising if He, in this respect as in others, should prove to be different from other elements. Judging from its molecular weight alone it would be expected to give a plot much nearer to the H-line than to the N-line, and lying between them.

Fig. 2 shows the relation between C and M^2, in which M is the molecular weight of chemistry. It is seen to be roughly a straight line, with equation $C = 0.058\ M^2 + 74$. The line has been drawn low among the points in order to give weight to the smaller values, which are perhaps the better. The coördinate M^2 was suggested by Sutherland's expression for C;

$$C = \frac{1}{c} m^2 f\left(\frac{1}{2a}\right)$$

This formula would not lead one to expect an intercept. If, however, Sutherland's value be equated to the empirical relation just given, we have

$$f\left(\frac{1}{2a}\right) = \frac{c}{h^2}\left(0.058 + \frac{74}{M^2}\right)$$

in which h is the mass in grams of a hydrogen atom, $c/2$ the conversion factor for kinetic energy and temperature, a the radius of a molecule and f the symbol for the law of potential energy due to the attracting forces.

Fig. 3, the relation between the coefficient K and the molecular weight, also shows a deviation from Sutherland's theory. According to that K should have the value

$$\frac{.064 c^{\frac{1}{2}} m^{\frac{1}{2}}}{(2a)^2}$$

while from the curve we obtain $K \cdot 10^7 = 0.593 M^{\frac{1}{2}} + 66$. Argon deviates about 4 per cent. from the line, and helium evidently does not form part of the series.

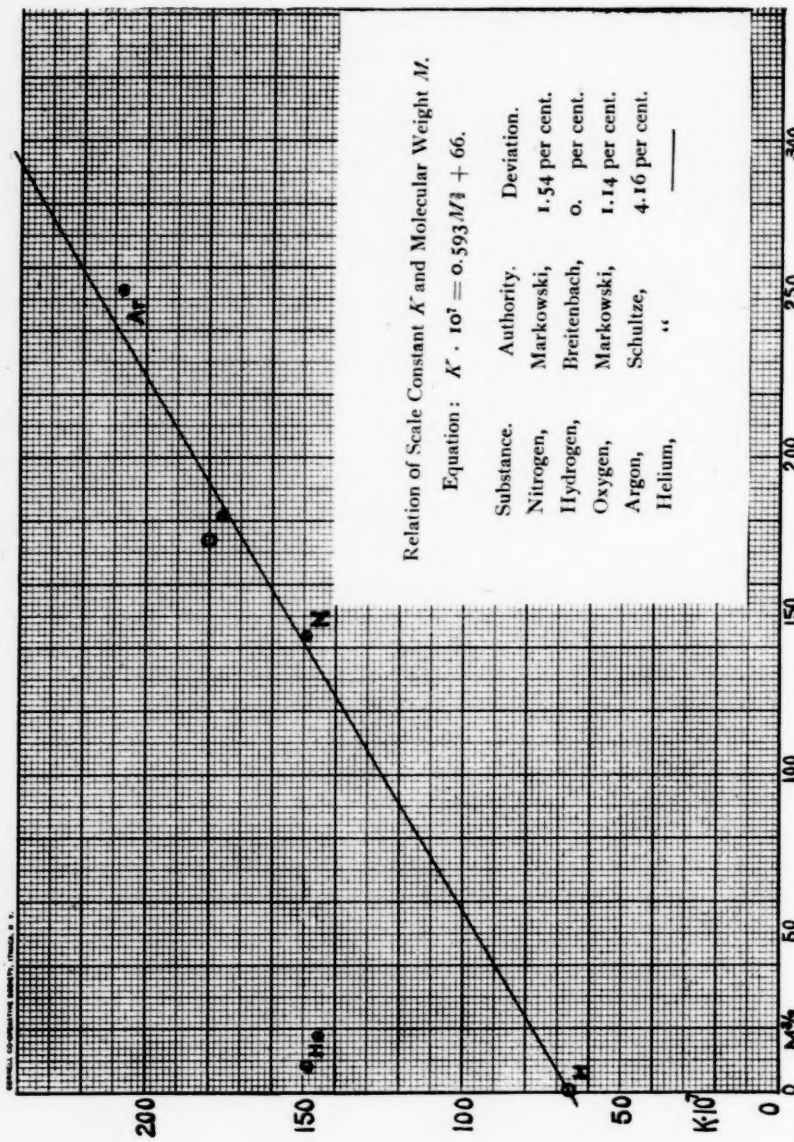

Fig. 3.

Fig. 4 is presented to show graphically the ratio between K and C, which is, as already shown, nearly constant for N, O and Ar.

The researches which may be supposed to be of promise as to the laws of viscosity of the elements would all be difficult; but if

the uncertainty relating to hydrogen and helium were cleared up, and information were secured as to the constants of neon, M = 20, chlorine, M = 71, and mercury vapor, M = 200, over large ranges of temperature, advances might result in our knowledge of the internal structure of elementary molecules.

Fig. 5, showing the curves for compounds, displays no such regularity as Fig. 1. The marked tendency for convergence shown by the curves for the elements is lacking, though there is a roughly outlined tendency for both K and C to increase (*a*) with increasing

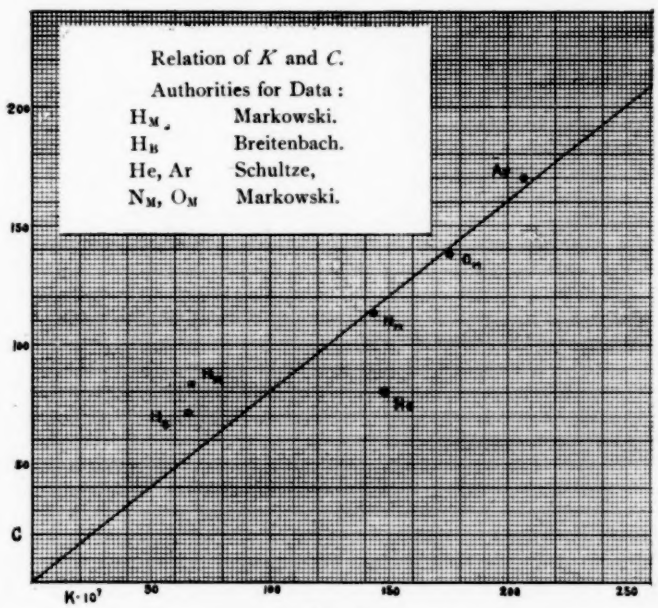

Fig. 4.

molecular weight, (*b*) with increasing molecular complexity and asymmetry. The comparative system of the curves for the elements may be due to the fact that their molecules are both more simple and more symmetrical than those of compounds. Since in the list of compounds presented no two can be said to be of either equal complexity or like symmetry, it is clear that among them we can expect to find no law indicated.

It is along these lines, then, that further research on the viscosity of gases should direct itself. Apparatus should be used which is

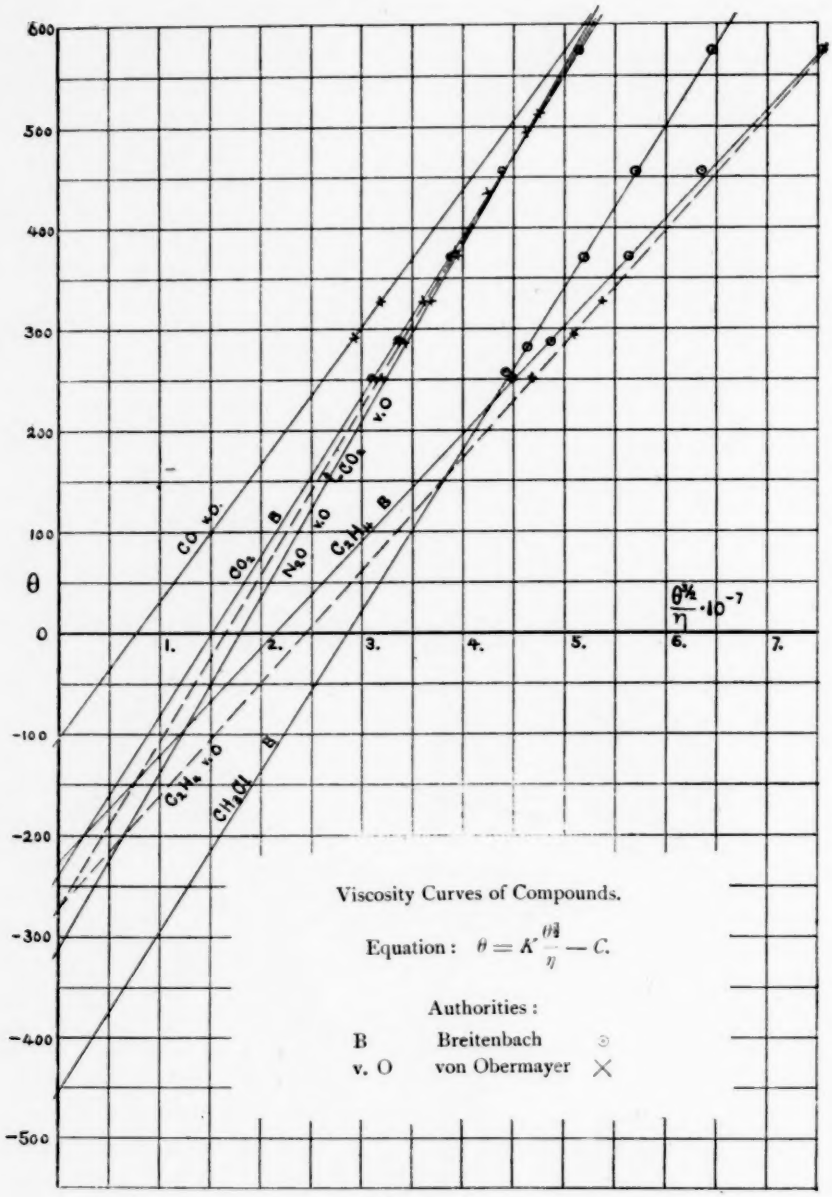

Fig. 5.

capable of giving results in absolute measure, so that both of the constants which determine the temperature change may be found

for the gases studied; and the gases selected should be (1) series of equal molecular weights but varying complexity of molecular structure; (2) series of equal molecular weights but varying symmetry of molecular structure; (3) series of different molecular weights but like structure.

In later papers the writer hopes to present discussions of Reinganum's equation, as applied to published data; on capillary calibration and certain other points in the construction of viscosity apparatus; and on the viscosity of certain gases which have theoretical interest but to which as yet little attention has been paid.

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An excellent and complete résumé of the development and literature of the subject of viscosity is to be found in a thesis by F. M. Pedersen, submitted to the Faculty of Science, New York University, 1905.

A METHOD FOR THE DETERMINATION OF ELECTROLYTIC RESISTANCE AND CAPACITY, USING ALTERNATING CURRENTS.

BY A. H. TAYLOR.

IF, according to Varley,[1] the polarization capacity is constant for small E.M.F., it may be offset by an inductance in series with the electrolyte, as has been done by Wien.[2] Or the resistance and capacity of the electrolyte may be balanced by a similar combination of resistance and capacity in the symmetrical arm of a bridge, as accomplished by Gordon,[3] or perhaps in a better way, by Scott.[4]

For great accuracy the methods of Gordon and of Scott require a wide range in the compensating capacity, and in the method of Scott it is advisable to have a smoothly variable capacity. Hence it would seem that Wien's method, where the capacity of the electrolyte is determined in terms of a smoothly variable inductance, is preferable.

The objection to Wien's method is that it is not independent of frequency, even if the electrolytic capacity be independent of frequency, and the object of the present paper is to propose a method of attacking this problem which will allow the use of a commercial alternating current, *i. e.*, a current of many overtones, hence of several frequencies.

In regard to its behavior toward alternating currents, an electrolyte which has polarizing electrodes may be regarded as a resistance in series with a capacity, a second resistance shunting both the resistance and the capacity in some cases.

It is possible to balance this combination in a bridge by inserting a shunted inductance in the same arm with the shunted electrolyte.

[1] Varley, Proc. Roy. Soc., 1871.
[2] Wien, Annalen der Phys., 1896.
[3] Gordon, Annalen der Phys., 1897.
[4] Scott, Annalen der Phys., 1899.

The figure gives the arrangement of the circuit, which consists of a bridge whose two proportional arms a and b are identical, and whose fourth arm consists of a variable non-inductive resistance R.

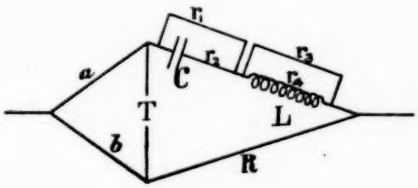

Fig. 1.

The third arm consists of a capacity C in series with a resistance r_2 and an inductance L of resistance r_4. As in the similar circuit devised by Trowbridge[1] for the differential transformer, the capacity and inductance must each be shunted in order to obtain a perfect telephonic balance, independent of the frequency.

The resistance r_3 shunts the inductance, while the resistance r_1 shunts the capacity + r_2.

Let c = the capacitance, $1/\omega C$, and x = the inductive reactance, ωL. The impedance of the third arm may be divided into two parts, Z_1 and Z_2, where

$$\frac{1}{Z_1} = \frac{1}{r_2 + jc} + \frac{1}{r_1}; \quad \frac{1}{Z_2} = \frac{1}{r_3} + \frac{1}{r_4 - jx}.$$

$$Z_2 = \frac{r_1(r_2 + jc)}{r_1 + r_2 + jc}; \quad Z_2 = \frac{r_3(r_4 - jx)}{r_3 + r_4 - jx}.$$

The bridge will be balanced when

$$Z_1 + Z_2 = R.$$

Substituting for Z_1 and Z_2, and multiplying by the conjugate complexes of the denominators,

$$R = \frac{r_1(r_2 + jc)(r_1 + r_2 - jc)}{(r_1 + r_2)^2 + c^2} + \frac{r_3(r_4 - jx)(r_3 + r_4 + jx)}{(r_3 + r_4)^2 + x^2}.$$

Separating into reals and imaginaries, and putting the imaginary parts equal to zero, since R has no imaginary component, we have

$$(1) \qquad R = \frac{r_1^2 r_2 + r_1 r_2^2 + r_1 c^2}{(r_1 + r_2)^2 + c^2} + \frac{r_3^2 r_4 + r_3 r_4^2 + r_3 x^2}{(r_3 + r_4)^2 + x^2}$$

and

[1] Trowbridge, PHYS. REV., 1905.

$$\frac{r_1^2 c}{(r_1+r_2)^2+c^2} = \frac{r_3^2 x}{(r_3+r_4)^2+x^2}. \tag{2}$$

Let $r_1 + r_2 = a$, and $r_3 + r_4 = b$. Then eq. (2) becomes

$$(b^2 + x^2) r_1^2 c = a^2 r_3^2 x + c^2 r_3^2 x,$$

or

$$(b^2 + x^2) r_1^2 = \frac{a^2 r_3^2 x}{c} + c x r_3^2.$$

Since $x = \omega L$, and $c = 1/\omega C$, we have

$$b^2 r_1^2 + \omega^2 L^2 r_1^2 = a^2 r_3^2 \omega^2 LC + \frac{L r_3^2}{C}.$$

This can be put in the form

$$C = \frac{L r_3^2 + \omega^2 (a^2 r_3^2 LC - L^2 r_1^2)}{b^2 r_1^2}. \tag{3}$$

If the balance is to be independent of the frequency, the term in ω^2 must disappear; that is, $a^2 r_3^2 LC = L^2 r_1^2$, or

$$C = \frac{L r_1^2}{a^2 r_3^2}. \tag{4}$$

If the term in ω^2 does disappear in (3), it becomes

$$C = \frac{L r_3^2}{b^2 r_1^2}. \tag{3}'$$

Combining this result with (4) we have,

$$\frac{L r_3^2}{b^2 r_1^2} = \frac{L r_1^2}{a^2 r_3^2}; \quad \text{or} \quad \frac{r_1^2}{r_3^2} = \frac{a}{b} = \frac{r_1 + r_2}{r_3 + r_4} \qquad \text{Eq. (B).}$$

Equation (3)′ thus may be thrown into the form

$$C = \frac{L}{ab} \qquad \text{Eq. (A),}$$

a form which is very easy to use in calculation.

Going back to equation (1) we may put it in the form

$$R(a^2 + c^2)(b^2 + x^2) = (b^2 + x^2)(r_1 r_2 a + r_1 c^2) + (r_3 r_4 b + r_3 x^2) a^2 + c^2)$$

Solving for c_1^2.

$$c^2 = \frac{(b^2 + x^2)(r_1 r_3 a - R a^2) + a^2 (r_3 r_4 b + r_3 x^2)}{R(b^2 + x^2) - r_1 (b^2 + x^2) - r_3 r_4 b + r_3 x^2)}. \tag{5}$$

Solving equation (2) for c^2 we have,

$$c^2 = \frac{(b^2 + x^2)r_1^2 c - a^2 r_3^2 x}{r_3^2 x}. \tag{6}$$

Eliminating c^2 from (5) and (6) we have,

$$\frac{(b^2 + x^2)r_1^2 c - a^2 r_3^2 x}{r_3^2 x} = \frac{(b^2 + x^2)(r_1 r_2 a - Ra^2) + a^2(r_3 r_4 b + r_3 x^2)}{(R - r_1)(b^2 + x^2) - (r_3 r_4 b + r_3 x^2)}.$$

Solving the equation for c,

$$c = \frac{r_3^2 x}{(b^2 + x^2)r_1^2}\left[\frac{(b^2+x^2)(r_1 r_2 a - Ra^2) + a^2(r_3 r_4 b + r_3 x^2)}{(R - r_1)(b^2 + x^2) - (r_3 r_4 b + r_3 x^2)}\right] + \frac{a^2 r_3^2 x}{(b^2 + x^2)r_1^2}$$

$$= \frac{r_3^2 x}{(b^2 + x^2)r_1^2}\left[\frac{(b^2 + x^2)(r_1 r_2 a - Ra^2) + a^2(b^2 + x^2)(R - r_1)}{(R - r_1)(b^2 + x^2) - (r_3 r_4 b + r_3 x^2)}\right]$$

$$= \frac{r_3^2 x}{r_1^2}\frac{[r_1 r_2 a - Ra^2) = a^2(R - r_1)]}{[(R - r_1)(b^2 + x^2) - (r_3 r_4 b + r_3 x^2)]}$$

$$= \frac{r_3^2 x}{r_1}\frac{(r_2 a - a^2)}{[(R - r_1)(b^2 + x^2) - (r_3 r_4 b + r_3 x^2)]}.$$

Since $c = 1/\omega C$, and $a = r_1 + r_2$,

$$\omega C = \frac{x^2(R - r_1 - r_3) + b^2(R - r_1) - r_3 r_4 b}{-x r_3^2(r_1 + r_2)}$$

$$= \frac{x(R - r_1 - r_3) + \dfrac{b^2(R - r_1) - r_3 r_4 b}{x}}{-r_3^2(r_1 + r_2)}.$$

Finally,

$$C = \frac{L(R - r_1 - r_3) + \dfrac{b^2(R - r_1) - r_3 r_4 b}{\omega^2 L}}{-r_3^2(r_1 + r_2)}.$$

If the balance is to be independent of the frequency, $b^2(R - r_1) = r_3 r_4 b$. Since b cannot equal zero, the only other relationship that will satisfy this condition is

$$b(R - r_1) = r_3 r_4 \quad \text{or} \quad R = r_1 + \frac{r_3 r_4}{r_3 + r_4} \quad \text{Eq. } (C).$$

Now equation (c) is simply the condition for steady current

balances, and might have been anticipated, since a balance independent of ω must hold for $\omega = 0$.

To summarize, the perfect balance condition means that

$$R = r_1 + \frac{r_3 r_4}{r_3 + r_4} \qquad (C)$$

$$\frac{r_1^2}{r_3^2} = \frac{r_1 + r_2}{r_3 + r_4} \qquad (B)$$

$$C = \frac{L}{ab} = \frac{L}{(r_1 + r_2)(r_3 + r_4)} \qquad (A)$$

If C is an ordinary condenser, and L a variable inductance, it is easy to obtain a perfect telephonic balance, such that these three equations are satisfied. In the case of the electrolyte, substituted for $C + r_2$, perfect balance is attained only for low current densities.

Equation (C) allows us to calculate r_1. If the calculated value is smaller than r_1, as observed, it will indicate a leaky condenser action.

Equation (B) allows us to calculate r_2, the *effective* electrolytic resistance.

Equation (A) is used for computing the capacity. The equation $(3)^1$, for capacity, is identical with Trowbridge's [1] equation (12) although this circuit contains the resistance r_2 which is not present in his circuit. The explanation is that his equation (12) is an approximation which is exceedingly close if $R - r_3$ is small compared to r_1. It is, however, impossible to realize this experimentally if very large capacities are to be compared with inductances which are under say, 40 millihenrys.

The order of precision is necessarily practically the same as for his method, when used for similar purposes.

The advantage is that capacities as large as 100 microfarads may be compared with relatively small inductances. It is applicable to the case of electrolytic capacity and resistance, and might possibly be used to advantage for the determination of condensers which have large absorption, since this is equivalent to r_2, the resistance in series with the condenser.

UNIVERSITY OF WISCONSIN,
November 29, 1906.

[1] Trowbridge, loc. cit.

FLUORESCENCE ABSORPTION IN RESORUFIN.[1]

By Frances G. Wick.

THE relation between absorption of light by a fluorescent substance and emission during fluorescence is difficult to understand. Waves of one length are absorbed while those of another length are emitted, in apparent contradiction to Kirchhoff's law. At least a step toward an explanation of this difficulty has been made in the experiments of Burke[2] and in the more recent work of Nichols and Merritt,[3] who found that a change takes place in the absorbing power during fluorescence. Previous observations of absorption had been made when the body was not under excitation. The work of Burke upon uranium glass, and that of Nichols and Merritt upon fluorescein in water, eosin in alcohol, and resazurin in alcohol, showed that during fluorescence a substance acquires the temporary power of absorbing the same wave-lengths which it emits. M. Camichel,[4] who has since made measurements of a similar kind, failed to find any change in absorbing power during fluorescence, throwing doubt upon the existence of the phenomenon.

The subject thus brought into question seemed of sufficient interest to justify further study. At the suggestion of Professors Nichols and Merritt the following series of tests for fluorescence absorption was made upon Weselski's diazo-rezorufin. Some experimental work[5] just completed, in which a study had been made of the fluorescence and ordinary absorbing power of resorufin, suggested this as a desirable substance for further investigation.

A preliminary set of experiments showed that resorufin acquires a measurable increase in absorbing power during fluorescence. This phenomenon was then studied in its relation to:

[1] A paper presented at the Ithaca meeting of the American Physical Society, June, 1906.

[2] Burke, John, Philosophical Transactions, Vol. 191A, p. 87, 1898.

[3] Nichols and Merritt, Physical Review, Vol. XIX., p. 397, 1904.

[4] Camichel, M., Comptes Rendus, 140, p. 139.

[5] Wick, Frances G., Physical Review, Vol. 24, p. 356, 1907.

1. Intensity of transmitted light.
2. Intensity of fluorescent light.
3. Thickness of absorbing layer.
4. Wave-length.
5. Concentration.

METHOD AND APPARATUS.

The method used was essentially the same as that employed by Nichols and Merritt, the instrument used being the Lummer-Brodhun spectrophotometer. The fluorescent solution, contained in a rectangular glass cell *R* (Fig. 1), was placed in front of slit *S*.

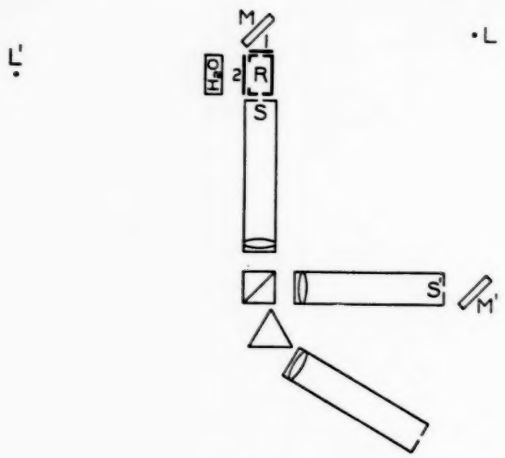

Fig. 1.

A bank of four acetylene flames *L'* was used as a source of fluorescence excitation. The source for transmission was an acetylene flame *L* from which light was diffusely reflected by a block of magnesium carbonate *M*. The intensity of transmission was adjusted by altering the angle of *M*. Screens 1 and 2 were arranged like shutters so that light from *M* or *L'* could be entirely cut off. For observations in which the intensity of fluorescence was varied, and those in which the thickness of the solution was changed at a constant wave-length, a glass cell containing a dilute solution of the fluorescent substance was placed between the flame *L* and the carbonate *M*. This acted as a screen to prevent fluorescence excitation by the transmitted light.

To determine the increase in absorption due to fluorescence three readings of the micrometer screw attached to S' were necessary.

1. *Transmission* (T). — Intensity of light transmitted from M, shutter 1 being open and 2 closed.

2. *Fluorescence* (F). — Intensity of the fluorescent light, shutter 2 being open and 1 closed.

3. *Transmission and Fluorescence combined* (C). — Intensity of light observed with 1 and 2 both open.

If the absorbing power were not changed during fluorescence the sum of the first two readings ($T + F$), would equal the third (C). This, however, did not prove to be the case. From actual observations

$$T + F > C.$$

An increase in absorption takes place which is measured by the expression

$$A_F = T + F - C.$$

This increase, A_F, is called by Nichols and Merritt "fluorescence absorption." The values of T, F, C and A_F were obtained in terms of the width of slit S' as read from the drum of its micrometer screw. Different sets of observations are not comparable since the comparison source was so adjusted as to produce the best possible match in color and intensity.

Difficulties and Sources of Error.

In all spectrophotometric work, especially determinations such as this, in which a small final result is obtained from the difference of two relatively large observations, the probability of error is great. This work was done in a dark room and special precautions were taken in the adjustment of screens and shutters to eliminate stray light. All possible care was taken in the arrangement of every part of the apparatus.

The most serious difficulty, however, was one of color match, and this could not be avoided by any mechanical adjustment. In making a set of observations extending through the fluorescence band the width of the collimator slit S (Fig. 1) had to be kept constant. A perfect color match was possible only when the intensities of the

two sources of light was such as to make slit widths S and S' (Fig. 1) equal. But as different wave-lengths were observed, keeping the comparison source L constant, the intensities of T, F and C, varied in such a way as to require the width of S' to be made in some cases greater and in others less than that of S.

Since the proper adjustment is difficult to make with accuracy it is essential that the observer's eyes be kept sensitive and that they be allowed to become perfectly restored after exposure to light before any observations are made. Care was taken in this work to have the periods of observations short, and results were rejected as inaccurate when the individual settings for any one measurement began to vary widely. An average of from three to six settings of the micrometer screw was taken for each of the values obtained.

FLUORESCENCE ABSORPTION AND INTENSITY OF TRANSMITTED LIGHT.

The first series of measurements was made to determine the relation between fluorescence absorption and the intensity of transmitted light. Tables I. and II. give the results obtained by a variation in the intensity of transmitted light, all other conditions being kept constant. Fluorescence absorption (A_F), it will be observed, is nearly constant for all values of transmission (T), in a given set of measurements.

According to Nichols and Merritt, as the intensity of the transmitted light increases from zero, fluorescence absorption must also increase to a certain maximum value, at which saturation is reached. At this point the intensity of transmitted light is so small as to make accuracy of measurement impossible. The tables here given serve as additional evidence of the truth of the conclusions drawn by Nichols and Merritt, that fluorescence absorption does not take place in accordance with the ordinary laws of absorption. An increase in the intensity of the incident light is not accompanied by a corresponding increase in absorption.

FLUORESCENCE ABSORPTION AND INTENSITY OF FLUORESCENCE.

In the next set of observations the intensity of the fluorescent light was varied by changing the distance of L' from the face of the cell (R) (Fig. 1).

TABLE I.

Dependence of Fluorescence Absorption upon the Intensity of the Incident Light. I., II. and III. indicate the Individual Observations from which the Average was Computed. The Measurements were made upon Concentration 1/32, *at Wave-length* .646 μ.

T				*F*				*C*				*T* + *F*	A_F
I.	II.	III.	Av.	I.	II.	III.	Av.	I.	II.	III.	Av.	Av.	Av.
25.2	24.8	24	24.7	12	11.4	11.3	11.57	34.3	34.5	33	33.93	36.27	2.34
36.4	36.5	35	35.97	11	11	11	11	44.5	44.7	44	44.4	46.97	2.57
56.2	60	59	58.4	11.3	10.8	11	11.03	65.4	66.5	66.2	66.03	69.43	3.4
68.3	67	67	67.43	10.6	10	11	10.53	75.5	75	76	75.5	77.96	2.46
74	73.8	74.8	74.2	10.6	11	11.2	10.93	82	82.8	82.4	82.4	85.13	2.71

TABLE II.

Dependence of Fluorescence Absorption upon the Intensity of the Incident Light. Concentration 1/128. *Wave-length* .646 μ.

T	*F*	*C*	A_F
13	10.4	21.73	1.67
31.5	10.1	39.56	2.04
68.23	9.93	75.2	2.96
89.9	10.43	97.8	2.53
98.27	10.3	105.375	3.195
	Concentration 1/64. *Wave-length* .646 μ.		
4.5	18.9	21.7	1.7
24.27	18.7	39.8	3.17
29.8	18.4	45.48	2.72
	Concentration 1/128. *Wave-length* .646 μ.		
13.17	10.5	21.43	2.24
32.85	10.5	39.4	3.95
57.47	10.87	62.5	5.84
68.65	10.5	76.96	2.19
84.3	10.6	90.7	4.2
114.1	11	117	8.1

The relation between fluorescence and fluorescence absorption was thus determined. The results indicate that for small values an increase in fluorescence causes an increase in fluorescence absorption. At a certain point saturation is reached, further increase in the intensity of fluorescence producing no change in fluorescence absorp-

tion. This is shown in Table III., graphically represented in Fig. 2. In the last two sets of observations given, there are no observations

TABLE III.

Dependence of Fluorescence Absorption upon the Intensity of the Fluorescent Light.
Concentration 1/64. *Wave-length* .646 μ.

T	F	C	A_F
30.2	3.4	33.2	.4
30.95	6.2	35.2	1.95
31.45	8.2	38.2	1.45
30.95	13.45	41.45	2.95
31.7	17.2	46.37	2.53
31.2	23.5	51.2	3.5
32.8	27.95	56.8	3.95
32.46	33.7	61	5.16
31.95	46.63	72.2	6.38
31.6	60.6	87.4	4.8
31.4	66.5	90.9	7
31.8	94.83	120.3	6.33
31.6	83.07	109.26	5.41
32.3	140.7	168.12	4.88
	Concentration 1/64. *Wave-length* .646 μ.		
34.9	7.37	41.1	1.17
33.82	9.37	40.25	2.94
32.9	12.9	42.9	2.9
34.9	16.27	46	5.17
35.7	20.7	51.5	4.9
35.33	22.53	53.37	4.49
34.63	23.7	52	6.33
	Concentration 1/128. *Wave-length* .646 μ.		
38.9	12.92	40.71	11.11
37.6	13.6	43.8	7.4
39.18	15.4	44.7	9.88
38.65	19	44.9	12.75
38.83	26	55.98	8.85
40.1	35	61.65	13.45
	Concentration 1/128. *Wave-length* .646 μ.		
45.88	10.05	44.125	11.805
44.9	13.37	46.17	12.1
44.78	19.6	47.66	16.72
46.3	22.95	54.9	14.35

below the point at which saturation is reached. This same saturation effect was previously observed by Nichols and Merritt.

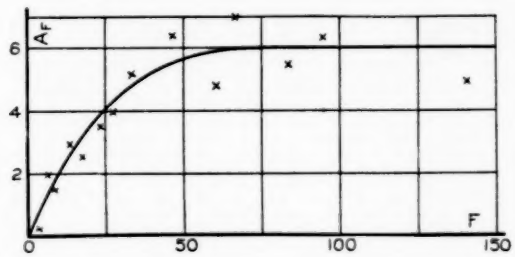

Fig. 2. Fluorescence absorption as a function of intensity of fluorescence.

Fluorescence Absorption With Variation in the Thickness of the Absorbing Layer.

In the previous experiments the section of the liquid excited to fluorescence through which the transmitted light passed was kept constant, *i. e.*, 5.45 cm. To find what effect a change in thickness has upon fluorescence absorption, the width of the section of the cell, *R*, exposed to the exciting light *L'* (Fig. 1) was varied. A section of the solution 1 mm. wide at the end of the cell next to the collimator slit *S* was first excited to fluorescence. The width of this section was gradually increased until it covered the whole face of the cell, readings being taken for each increase in width.

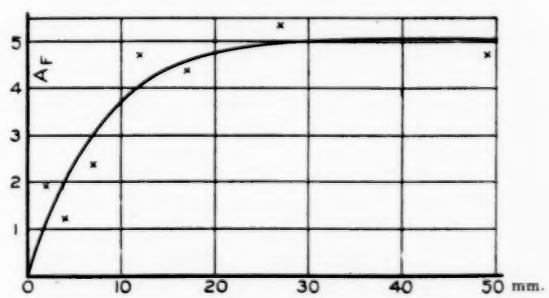

Fig. 3. Fluorescence absorption as a function of the thickness of the absorbing layer.

Fig. 3 shows the result. Fluorescence absorption increases with an increase in thickness up to a certain point, for this particular concentration about 1.25 cm. Further increase in thickness pro-

duces no change in fluorescence absorption. The complete set of curves from the first observations given in Table IV. is shown in Fig. 4.

As a check upon this work the same series of measurements was made beginning with the 1 mm. slit at the opposite end of the

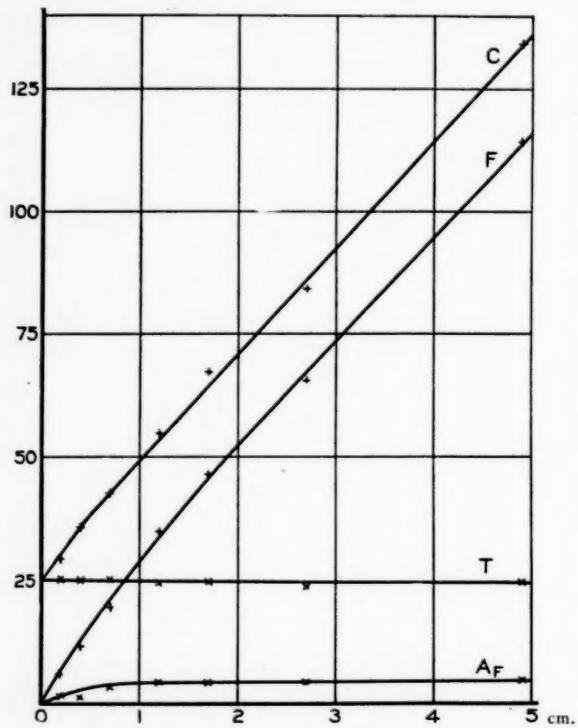

Fig. 4. Curves showing the effect of change in the thickness of the absorbing layer, the width of band being increased from the end of the cell next to the collimator tube toward the end next to the source of transmitted light.

F = observed fluorescence.
T = transmission.
C = fluorescence and transmission combined.
$A_F = T + F - C$ = fluorescence absorption.

cell, *i. e.*, the end next to M (Fig. 1), and gradually increasing the width of the opening toward the collimator tube until the whole face of the cell was excited. The results, given in the third set of observations in Table IV., and Fig. 5, were similar to those ob-

TABLE IV.

Dependence of Fluorescence Absorption upon the Thickness of the Absorbing Layer.
Concentration 1/64. *Wave-length* .646 μ.

Variation in thickness beginning at end of cell next to the collimator tube.

Thickness.	T	F	C	A_F
2 mm.	25.5	6.1	29.69	1.91
4	25.21	11.85	35.83	1.23
7	25.43	19.58	42.65	2.36
12	24.6	35	54.91	4.69
17	25.05	46.57	67.25	4.38
27	23.9	65.7	84.26	5.34
49	24.7	114.2	134.2	4.7

Concentration 1/64. *Wave-length* .646 μ.

1 mm.	28.9	3.95	31.94	.91
3	28.9	8.4	34.1	3.2
5	28.1	12.2	37	3.3
7	28.1	16	40.2	3.9
12	27.84	24.3	47.3	4.84
17	28.1	34.8	57.3	5.6
27	27.5	46.88	68.74	5.64
37	28.1	62.8	84.5	6.4

Concentration 1/64. *Wave-length* .646 μ.

Variation in thickness beginning at end of cell away from the collimator tube.

1 mm.	27.5	2.6	29.4	.7
3	27.6	6	32.1	1.5
5	27.63	8	33.6	2.03
7	26.58	12.8	35.3	4.08
12	26.9	20.37	42.7	4.57
17	26.2	28.45	49.6	5.05
22	26.2	37.3	54.45	9.05
32	26.2	46.9	67.2	5.9
42	26.2	59.7	80.7	5.2
54	26.2	71.5	91.8	5.9

tained from measurements in the other direction. This might be expected since the solution used was very dilute and at the wave-length studied ordinary absorption was insignificant.

In Fig. 5, the curve marked A_F shows the same saturation effect observed in Figs. 3 and 4. Curve F, Figs. 4 and 5, gives the observed fluorescence and curve C the fluorescence and transmis-
sio n combined. Curve F bends to about the point at which satu-

ration is reached in A_F then becomes a straight line. This is explained by the fact, that, at first, with increased thickness there is an increase in the fluorescence absorption. This decreases the intensity of the observed fluorescent light. After saturation has been reached the increase in the observed fluorescence is proportional to the increase in thickness, fluorescence absorption entering

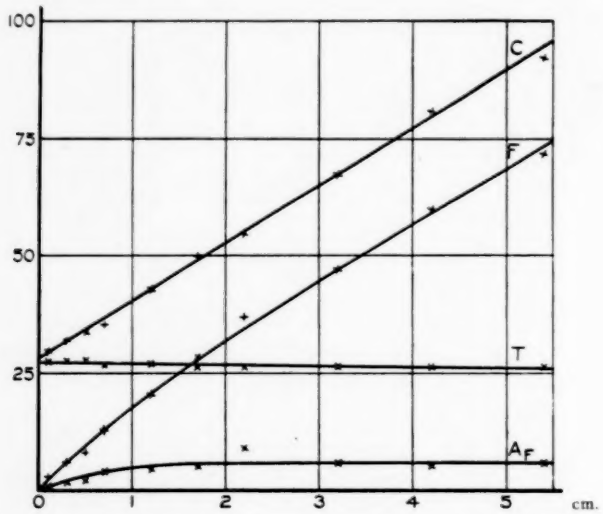

Fig. 5. Curves showing the effect of change in thickness of the absorbing layer, the thickness being increased from the end of the cell next to the source of transmission toward the collimator tube.

as a constant factor. Curve C is approximately a straight line. Other sets of measurements were made with similar results. In every case a saturation point was reached beyond which an increase in thickness produced no change in fluorescence absorption.

It thus appears that the energy absorbed from the transmitted light increases with increasing thickness up to a certain limit, beyond which fluorescence absorption is not changed by increasing the thickness of the fluorescent solution. A saturation point is reached similar to that observed in changing the intensity of fluorescence.

RELATION OF FLUORESCENCE ABSORPTION TO WAVE-LENGTH.

The relation between fluorescence absorption and wave-length was determined by shifting the telescope into different positions in the

fluorescence band and making readings of T, F, and A_F at different wave-lengths.

TABLE V.

Dependence of Fluorescence Absorption upon Wave-length. Concentration 1/16.

Wave-length.	T	F	C	A_F
.542 μ	.7	1.7	1.5	.9
.562	.38	2.3	2.55	.13
.589	7.76	15.75	22.9	.61
.614	31.97	26.31	54.7	3.58
.646	39.88	13.8	52.7	.98
.69	36.55	7.82	43.35	1.02
	Concentration 1/2.			
.589 μ	.55	4.325	4.9	.025
.614	11.3	20.63	29.575	2.355
.646	31.75	27.95	51.975	7.725
.69	40.4	15.9	52.375	3.92

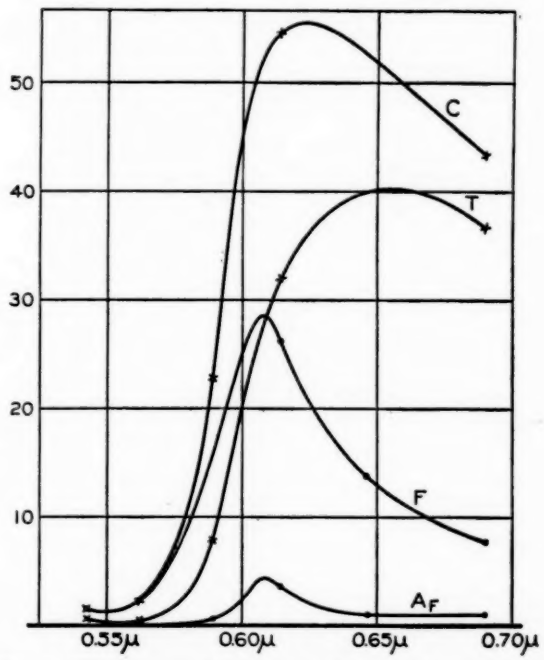

Fig. 6. Fluorescence absorption as a function of wave-length.

Table V., represented graphically in Fig. 6, gives the average of three sets of observations made upon concentration one sixteenth,

a screen of one sixty-fourth resorufin being so adjusted between L and M (Fig. 1) as to cut off those rays which might produce fluorescence and thus introduce error. The fluorescence absorption curve, A_F, follows the fluorescence curve, F, the maxima of the two corresponding in wave-length. Table V. gives also the average of three sets of observations made upon concentration one half.

Other measurements given in Table VI. were made earlier without the interposition of the resorufin screen. These indicate greater errors of observation but in general it will be noticed that the greatest fluorescence absorption takes place in the region of greatest fluorescence.

TABLE VI.

Dependence of Fluorescence Absorption upon Wave-length. Concentration 1/128.

Wave-length.	T	F	C	A_F
.524 μ	11.7	2.5	11.8	2.3
.542	11.56	2.7	11.73	2.43
.562	9.86	4.55	13.35	1.06
.589	14.5	16.6	29.75	1.35
.614	23.65	17.9	37.25	4.3
.646	32.9	10.4	39.35	3.95
	Concentration 1/64.			
.524 μ	10.25	1.95	10.3	1.9
.542	7.73	1.8	7.5	2.03
.562	6.47	5.07	9.27	2.27
.589	8.6	35.9	42.4	2.1
.614	21.73	47.7	65.23	4.2
.646	30.83	26.9	53.97	3.76
	Concentration 1/128.			
.542 μ	11.2	1.2	11.45	.95
.562	7	4.2	10.2	1
.589	14.45	29.8	40.7	3.55
.614	34.4	39.7	64.9	9.2
.646	46.6	22.5	55.4	13.7
.69	48.2	19.45	59.8	7.85

CHANGE IN CONCENTRATION.

The relation between fluorescence and fluorescence absorption is further brought out by a comparison of the results obtained from different concentrations. The positions of maximum fluorescence and fluorescence absorption correspond in each case.

Fig. 7 gives results from two concentrations. Curves F and A_F represent fluorescence and fluorescence absorption for concentration one sixteenth, the corresponding maximum being .61 μ. Curves F' and A_F' represent fluorescence and fluorescence absorption of concentration one half. The maximum of fluorescence, F', has shifted

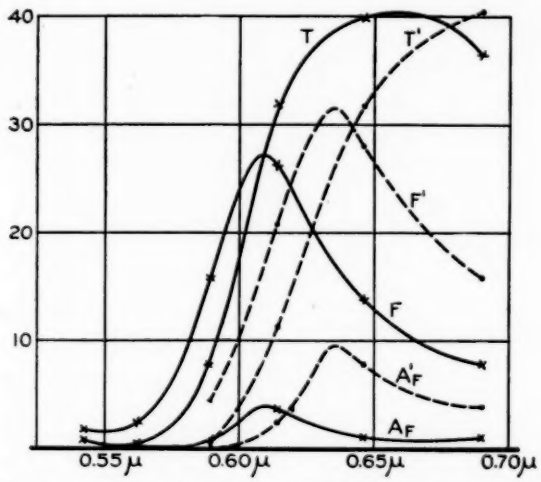

Fig. 7. Fluorescence absorption as a function of wave-length in different concentrations.

Curves T, F and A_F correspond to concentration $\frac{1}{16}$.
Curves T', F', and A_F' correspond to concentration $\frac{1}{2}$.

to about .635 μ—an effect due to increased absorption on the side of the band next to the violet. The maximum fluorescence absorption, A_F', has also shifted to about the same point.

Conclusion.

The experiments described in this paper serve as a confirmation of the work done by Professors Nichols and Merritt for whose encouraging interest the author wishes to express her grateful appreciation.

The fact that during fluorescence a substance acquires a temporary power of absorbing those rays emitted during excitation seems reasonable; but certain phenomena observed in connection with this increase baffle explanation. It seems natural to expect that with an increase in the intensity of transmitted light an increase in fluores-

cence absorption will take place according to the known laws of ordinary absorption, but such is not the case. The increased absorption seems independent of the intensity of transmitted light.

The saturation effect observed with increase in the thickness of the fluorescent solution is equally strange. After a certain thickness has been reached further increase in thickness has no effect upon fluorescence absorption.

Another surprising fact brought out is that the fluorescence absorption band corresponds in position with the *observed* fluorescence band. It might be expected that a phenomenon which seems to be an accompaniment of fluorescence should show a maximum effect not at the observed maximum of fluorescence, after absorption has taken place, but at the actual maximum fluorescence point found by correcting for absorption.

In this paper no attempt has been made at explanations, the object being simply to present the observational facts with the object of discovering whether or not any peculiar absorption phenomena accompany fluorescence in resorufin.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY.

REFLECTION AND TRANSMISSION OF ELECTRIC WAVES BY RESONATOR GRATINGS.

Remarks on a Paper Under This Title, by F. C. Blake and C. R. Fountain.

By Clemens Schaefer.

IN a research,[1] which has just been published, Messrs. Blake and Fountain have investigated the reflection and transmission of electric waves by systems of resonators.

The method which they use is similar to that which Garbasso,[2] Aschkinass and I[3] have employed; we will speak later of the changes they have introduced. Their results confirm in part those previously obtained by the above-mentioned authors. Along with other work, I[4] published some of their results in an article under the title, "Concerning the Selective Properties of Resonator Gratings," in the Annalen der Physik, in January, 1905. This article has apparently not come to the notice of Messrs. Blake and Fountain. At least I can find no evidence that it has, although in the article by Pätzold, cited by them in their appendix, my work is frequently mentioned.

This is my reason for giving the work of Messrs. Blake and Fountain somewhat closer consideration, and especially since their method, in those points where it differs from that of Aschkinass and myself, is by no means an improvement, but to my mind at least is even poorer.

As Garbasso first showed, the resonator gratings here used possess selective properties for electric waves; *i. e.*, they reflect to a considerable extent only those waves whose period of vibration

[1] F. C. Blake and C. R. Fountain, Phys. Rev., XXIII., p. 257, 1906.

[2] A. Garbasso, Atti Acc. di Torino, XXVIII., pp. 470 and 816, 1893. Journal de Physique, XXII., p. 259, 1893.

[3] E. Aschkinass and C. Schaefer, Ann. d. Phys., V., p. 489, 1901.

[4] C. Schaefer, Ann. d. Phys., XVI., p. 106, 1905.

does not differ greatly from their own. Two methods present themselves for the study of the relation between reflection, with its corresponding transmission, and the wave-length:

I. We may use a *single resonator grating*, which remains unchanged during the experiment. In this case we must use a variable exciter and receiver to give us the necessary wave-lengths within the limits over which we wish to study the extent of reflections or transmission. This method is exactly that which is used in the corresponding measurements in optics. This method seems to me for certain reasons to have the advantage over the one to follow.

II. We work with a *single wave-length, i. e.*, with an unchanging exciter and receiver, varying instead the resonator gratings, *i. e.*, the length of the resonators. With this method, too, which in a certain sense is only a reversal of the previous one, we may obtain unimpeachable results. By this method, for example, the investigations of Garbasso, and Aschkinass-Schaefer were carried out. Messrs. Blake and Fountain have also used this second method, but in a way that is not entirely free from objection.

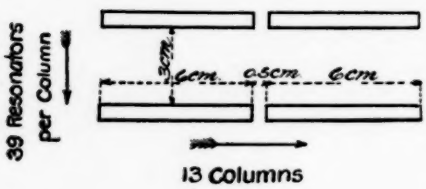

Fig. 1.

I will now try to make this clear. The characteristic point in the method used is the variation of the resonator grating. Let us see how this was done in this investigation. According to the table, page 265, the authors start out, for example, in their curves T and R of Fig. 8, with a grating, the structure of which can be seen from the accompanying figure.

This grating they change by the gradual decrease of the resonators, *all other dimensions of the apparatus remaining unaltered.*

There are objections to this method, although at first sight it is apparently correct. In order to make this clear, consider the following: The power of a resonator grating to reflect, and consequently too, to transmit, depends not only on the length of the resonators but also on their number and their arrangement. So gratings made of resonators of different lengths are in general not at all comparable. That is, the question which rises in using the

second method is this: How must resonator gratings, consisting of resonators of different lengths, be arranged, in order to be comparable with another? The answer to this question is easy, if we for a moment consider the relations holding when we employ the first method, which, as already stated, is entirely analogous to the method used in the corresponding optical measurements. Here the grating stays unchanged during the experiment, while the wave-length is varied. If we pass from a wave-length λ_1 to a second λ_2, the corresponding linear dimensions of the grating have changed in the proportion of λ_2 / λ_1, using as a unit the new wave-length λ_2. And here we see the answer to our question: In using the second method it is not sufficient to change the length of the resonators alone, but all corresponding dimensions of the grating must be changed in the same ratio. Only thus can we obtain comparable relations, as in the case of those optical experiments where we meet selective reflections, and in this way we get exactly these relatively simple relations. The error in the method of Blake and Fountain is that they did not notice this fact, although Aschkinass and I laid especial emphasis upon it.

This explanation makes it at once clear how much simpler the first method is, because this particular difficulty does not enter into it at all. Hence the curves of Messrs. Blake and Fountain are quite complicated, and cannot well be compared with those of the previous investigators, and it is not possible to apply the ordinary dispersion formulæ to them directly as would be the case with the curves of Aschkinass and myself. In short, Blake and Fountain obtained, on account of the construction of their gratings, complicated relations instead of relatively simple ones.

I have mentioned already that many of the results obtained by Messrs. Blake and Fountain are only substantiations of the work of previous authors. I will now make this clear in a special case. Blake and Fountain discuss this problem: How does the length of the resonators corresponding to maximal resonance change, when the elements of the grating are brought closer together? They find, that a diminution of the distance between the resonators in a direction at right angles to the electric force requires a greater resonator length to produce maximal resonance. According to their results,

within the limits investigated, a diminution of the distance in a direction parallel to the vector of electric force makes no difference at all. From the standpoint of the first method, this means that in the first case the point of maximal resonance is changed so that it occurs at a smaller wave-length, while in the second case it is unaffected.

I discussed this same question in the article previously mentioned, and studied the phenomena by the first method, which seems certainly better on account of its simplicity. The results I obtained are these: (1) A diminution of the distance in a direction at right angles to the electric vector changes the maximal resonance to a point of smaller wave-length. This is the same result as obtained by Blake and Fountain. (2) A decrease of the distance in a direction parallel to the electric force changes the maximal resonance to a point of greater wave-length. This result is exactly opposite to that of Blake and Fountain; however this contradiction is easily explained, since I varied the distances within wider limits than they. Further it must not be forgotten that, strictly speaking, the curves of Blake and Fountain are not at all comparable with mine. The fact that they did not find the change in the point of maximal resonance in the second case, is of itself a proof of the unsuitability of their method. Moreover, according to the explanation which Blake and Fountain put forward for the change in the first case, one would also expect a change in the second. Consider this explanation for a moment. They say, page 268: "that the resonance length should decrease with increasing distance between the strips was to be expected; for since at any instant the currents in the resonators are all in the same direction, the effect of induction is reduced by decreasing the distance between them, hence in order to respond to a given wave the capacity of the resonators must be increased, that is, they must be lengthened."

I cannot understand this explanation because of its inexactness o statement. What is meant by "the effect of induction"? The only factor concerning the induction, which can be changed through a change in the distance between the resonators, is the coefficient of mutual induction. As I previously showed,[1] however, this change

[1] C. Schaefer, l. c.

would, in the first case, have an effect exactly opposite to the observed; while in the second case, on the other hand, the theoretical changes would correspond with those observed by myself. These points make it clear that the explanation in the first case, by means of the coefficient of mutual induction, is not complete. We must rather take into account that a change in the distance between the resonators also changes in general their capacity. The change in capacity, however, causes in both cases a change in the point of maximum resonance corresponding to the experimental results. That is, the explanation of Messrs. Blake and Fountain is certainly, if not absolutely wrong, at least incomplete.

I pass now to the consideration of a phenomenon which Messrs. Blake and Fountain discovered, and to which they gave the name of "Extra transmission." They met this phenomenon in the case of resonator gratings glued to glass plates. Essentially it is this, that such a resonator grating glued to glass, under certain conditions, *transmits more*, and consequently *reflects* less, than the glass plate alone. The authors explain this remarkable phenomenon by saying that the glass plate and resonators may be a medium with a variable index of refraction, and that, according to the optical dispersion formulæ, under certain conditions the index of refraction may become smaller than that of the glass itself. Although, as I have already shown, the optical dispersion formulæ are not here applicable entirely without restriction, still there is no objection to be raised to this explanation, *provided the phenomenon in question is a fact.* I must say, however, that the description of Blake's and Fountain's experiments has not convinced me of the reality of the phenomenon. My reasons are: first, that none of the earlier experimenters found this phenomenon, although, according to Blake and Fountain, it is quite marked. In particular, Aschkinass and I did not find this to happen. Hence in this latter investigation we must assume quite large errors of observation. Now, such large errors are *surely* here out of the question, as is easily seen from the excellent correspondence of our measurements of the dielectric constant with those of the other investigators. Moreover, I have repeated the measurements of Blake and Fountain in question, reproducing their arrangement of apparatus. The only change I made was to

restrict the wave-flow by making it pass through two diaphragms, with openings of 24 × 32 cm. The resonator grating used was of the same size. It only remains to be said that the area of the resonator grating was *smaller* than the mouth of the concave mirror used. With this apparatus I was likewise unable to find any "extra transmission." More than that, the grating always transmitted less energy than the glass plate alone. If the diaphragms were removed, the transmittivity of the resonator grating *increased;* if the rays were made *divergent*, by moving the exciter a bit from the focus of the concave mirror, the transmittivity *increased still more*. Now this result is an *approximation* of the phenomenon of "extra transmission"; but I did not succeed in verifying the observations of Blake and Fountain.

The question is, must we accept the explanation of Blake and Fountain, or can we give another. To answer this, it must be kept in mind, that these authors did *not use diaphragms* to restrict the wave flow; moreover the area of their resonator grating was larger than that of their parabolic mirror. Now, if we assume that the original electric waves were not exactly parallel, but slightly *divergent*, it would actually be possible to observe a phenomenon like "extra-transmission." For the individual resonators of the grating would be set in vibration by the electric wave striking them, and thus would become themselves centers of spherical waves going out in all directions. Now, since in the experiments of Blake and Fountain the grating had a larger area than the concave mirror, in the focus of which the receiver is placed, "*side waves*" from the resonators on the border of the grating could reach the receiver when the grating was interposed, while the interposition of the glass plate alone let only direct waves reach the receiver. Under certain conditions these "side waves" may become so marked, that putting the grating in the path of the wave would actually give an *increase* of the amount of energy reaching the receiver. As a matter of fact, M. Paetzold observed this very thing in the case of *divergent* radiation without interposition of diaphragms with gratings, whose area was larger than the mouth of the mirrors he used. This is the explanation of the results of my repetition of the experiments of Blake and Fountain. As soon as the diaphragms are taken away,

we find an increase of transmittivity, which is, and ought to be, under proper experimental conditions, *constant.* The increase above the normal is still greater if we intentionally make the paths of the radiation divergent.

I do not yet say, that the "extra transmission" of Blake and Fountain is likewise due to poor experimental arrangements. But so long as the above objections to the experimental arrangement of the authors are not refuted, there is no reason for the assumption of a *new phenomenon.*

BRESLAU, PHYSICAL LABORATORY OF THE UNIVERSITY,
January, 1907.

THE EFFECT OF A TRANSVERSE MAGNETIC FIELD ON METALLIC RESISTANCE.

BY E. P. ADAMS.

THE effect of a transverse magnetic field in changing the resistance of a conductor has been investigated by many observers; Patterson,[1] and very recently Grunmach [2] have made very careful measurements of this effect in a number of metals. Considering the size of the effect, the agreement between the results of these two observers is, on the whole, very good. In all the metals which have been tried, with the exception of the ferromagnetic metals, iron, nickel, and cobalt, a transverse magnetic field is found to produce an increase in the resistance. Grunmach found that in the case of the ferromagnetic metals, weak magnetic fields produced an increase in the resistance. On increasing the magnetic force the effect became smaller, until for a certain value of the magnetic force it disappeared; for greater values the resistance was diminished. With one specimen of iron, and with nickel, the effect appeared to be a decrease of the resistance from the start.

A theory of the effect of a transverse magnetic field in increasing the resistance of conductors has been given by J. J. Thomson.[3] He assumes that the electric current is carried by free corpuscles, which, when no external forces act, are in motion in all directions, just as the molecules of a gas inside a closed vessel. The corpuscles are, in fact, assumed to behave just as the molecules of a gas, with this exception; the collisions which take place are not between corpuscles and corpuscles, but between corpuscles and atoms of the metal. When an electric force is applied, a drift of the corpuscles in one direction is produced, which is the electric current. In this explanation, in order to get an effect of the right sign, it is necessary to assume that the collisions between corpuscles and atoms are greatly influenced by the electric charges carried by the former.

[1] Philosophical Magazine, 3, p. 643, 1902.

[2] Annalen der Physik, 22, p. 141, 1907.

[3] Rapports presentes au Congres International de Physique, III., p. 138, 1900. Philosophical Magazine, 3, p. 353, 1902.

The assumption of collisions like those between hard elastic spheres cannot therefore be made. This theory can be made to apply to the ferromagnetic metals only by assuming that in them the collisions are of an altogether different character from the collisions in other metals. Lorentz,[1] on the other hand, expressly assuming collisions uninfluenced by the electric charges of the corpuscles, has successfully applied this theory to the thermoelectric properties of metals; it thus seems as if the assumption of collisions between corpuscles and atoms like those between hard elastic spheres ought also to explain the change in resistance of conductors in a magnetic field.

In the present paper it will be shown that, keeping to the assumption of collisions of this nature between corpuscles and atoms, and introducing the probable effect of the magnetic field in changing the molecular configuration of the metal, it is possible to explain the sign of the effect in the ordinary para- and diamagnetic metals, and in the ferromagnetic metals, as well. The theory of the effect as given by J. J. Thomson will first be developed, and afterwards this theory will be modified by introducing the effect of the magnetic field on the molecular configuration of the metal.

We assume that the corpuscles are all of one kind and move uniformly in all directions with the velocity U. In the paper of Lorentz, referred to above, the theory of electric and thermal conductivity has been given where the distribution of velocities among the corpuscles is assumed to be that given by Maxwell's law for the distribution of velocities among the molecules of a gas. But for the present purpose, since we can hope to get an idea of the order of magnitude only of the effect, this complication is unnecessary.

In a magnetic field the velocity of the corpuscles moving in a plane perpendicular to the magnetic force will be altered. Taking the xy plane as the plane of motion, the current being in the x-axis, and the magnetic force in the z-axis, the equations of motion of a single corpuscle are:

$$\left.\begin{aligned} m\frac{\partial^2 x}{\partial t^2} &= eX - He\frac{\partial y}{\partial t} \\ m\frac{\partial^2 y}{\partial t^2} &= He\frac{\partial x}{\partial t} \end{aligned}\right\} \qquad (1)$$

[1] Archives Neerlandaises des Sciences Exactes et Naturelles, 10, p. 336, 1905.

m being the mass, e the charge, and H the intensity of the uniform magnetic field.

From this we get:

$$m\frac{\partial y}{\partial t} = Hex$$

since when $x = 0$,

$$\frac{\partial y}{\partial t} = 0.$$

As the effect produced by the magnetic field is known to be small it will be permissible to take for x in this equation the value it would have if no magnetic field were applied; *i. e.*:

$$x = \frac{1}{2}\frac{e}{m}Xt^2 + Ut$$

$$\frac{\partial y}{\partial t} = \frac{He}{m}\left(\frac{1}{2}\frac{e}{m}Xt^2 + Ut\right).$$

Substituting this in the first of (1):

$$\frac{\partial^2 x}{\partial t^2} = \frac{e}{m}X - \frac{H^2e^2}{m^2}\left(\frac{1}{2}\frac{e}{m}Xt^2 + Ut\right)$$

and hence:

$$\frac{\partial x}{\partial t} = \frac{e}{m}Xt - \frac{H^2e^2}{m^2}\left(\frac{1}{6}\frac{e}{m}Xt^3 + \frac{1}{2}Ut^2\right) + U.$$

Calling $\bar{u}$ the mean velocity of the corpuscle due to the electric and magnetic field between two collisions, and T the time of free path, we get:

$$\bar{u} = \frac{1}{T}\int_0^T \frac{\partial x}{\partial t}\,dt = \frac{1}{2}\frac{e}{m}XT - \frac{H^2e^2}{m^2}\left[\frac{1}{24}\frac{e}{m}XT^3 + \frac{1}{6}UT^2\right] + U. \quad (2)$$

If there are n corpuscles per unit volume, the current in the direction of the x-axis will be:

$$I = ne\bar{u} = \frac{1}{2}\frac{e^2}{m}nXT - \frac{1}{6}\frac{H^2e^3}{m^2}\left[\frac{1}{4}n\frac{e}{m}XT^3 + nUT^2\right]. \quad (3)$$

The last term of (2) drops out because there will be as many corpuscles for which U is positive as negative. We cannot, however, say that the second term in the brackets in (2) drops out, since,

owing to the factor T^2, this will be different for corpuscles whose initial velocities are in opposite directions. A corpuscle for which U is negative will have a greater value of T, as a result of the electric force, than one for which U is positive. This term can be easily evaluated if we consider the collisions between corpuscles and atoms to be like those between hard elastic spheres. For half of the corpuscles U will be negative; so this term can be written:

$$\frac{1}{2} nUT^2 - \frac{1}{2} nUT'^2 = nUT\Delta T,$$

T being the time between collisions when no electric force is applied. If L is the free path of a corpuscle:

$$\Delta T = T - T' = L\left\{ \frac{1}{U + \frac{1}{2}\frac{e}{m} XT} - \frac{1}{U - \frac{1}{2}\frac{e}{m} XT} \right\}$$

so that:

$$nUT^2 = -\frac{e}{m} nXT^3$$

and the electric current is given by the expression:

$$I = \frac{1}{2}\frac{e^2}{m} nXT + \frac{1}{8}\frac{H^2e^3}{m^2}\frac{e}{m} nXT^3.$$

The first term on the right is the current when no magnetic force is applied. The effect of the magnetic force is thus to increase the current, or decrease the resistance, which is contrary to observed facts.

J. J. Thomson, however, considers that the collisions between corpuscles and atoms are strongly influenced by the electric charges carried by the corpuscles. Without going into any calculation it is easy to see that if this influence is sufficiently pronounced, the difference, ΔT, between the time of the free path in the direction of the current and the opposite direction, will be very small; hence the second term in the brackets of (3) may be neglected in comparison with the first term, and we get:

$$I = \frac{1}{2}\frac{e^2}{m} nXT - \frac{1}{24} H^2 \frac{e^4}{m^3} nXT^3.$$

This indicates an increase in resistance due to the magnetic field.

Now if we suppose that a magnetic field produces a rearrangement of the molecules within a metal, it seems very reasonable to assume that the free path of a corpuscle will be altered. Without saying anything definite as to the nature of this change, let us assume that a magnetic field, H, produces a change, δT, in the time between two collisions of a corpuscle. δT will then, in general, be some function of H. With no magnetic field the current will be:

$$I = \frac{1}{2}\frac{e^2}{m} nXT. \tag{4}$$

In a transverse magnetic field the current will be, by (3):

$$I' = \frac{1}{2}\frac{e^2}{m} nXT' - \frac{1}{6}\frac{H^2e^3}{m^2}\left[\frac{1}{4} n \frac{e}{m} XT'^3 + nUT'^2\right] \tag{5}$$

where T is changed to T' the free time in the magnetic field. The effect of the magnetic field will therefore be:

$$I - I' = \frac{1}{2}\frac{e^2}{m} nX\delta T + \frac{1}{6}\frac{H^2e^3}{m^2}\left[\frac{1}{4} n \frac{e}{m} XT'^3 + nUT'^2\right]. \tag{6}$$

As we suppose the change in T produced by the magnetic field to be small, we can put:

$$T'^3 = T^3 - 3T^2\delta T,$$
$$T'^2 = T^2 - 2T\delta T.$$

Then by bringing in the assumption that the collisions between the corpuscles and atoms are like those between hard elastic spheres, we find, by the method previously employed:

$$I - I' = \frac{1}{2}\frac{e^2}{m} nX\delta T + \frac{1}{6} H^2 \frac{e^3}{m^2}\left[-\frac{3}{4} n \frac{e}{m} XT^3 + \frac{1}{4} n \frac{e}{m} XT^2\delta T\right]. \tag{7}$$

The second term in the brackets will be small compared to the first, and we thus have:

$$I - I' = \frac{1}{2}\frac{e^2}{m} n X\left[\delta T - \frac{1}{4} H^2 \frac{e^2}{m^2} T^3\right]. \tag{8}$$

If R is the resistance of the conductor with no magnetic field, and R' the resistance in the magnetic field, we have:

$$I - I' = \left(\frac{1}{R} - \frac{1}{R'}\right) X$$

or

$$\frac{\delta R}{R} = \frac{\delta T}{T} - \frac{1}{4} H^2 \frac{e^2}{m^2} T^2. \tag{9}$$

In order that the magnetic field may increase the resistance, it is necessary, first, that δT be positive; this means that a change in molecular configuration arising from the magnetic field produces a diminution in the time between two collisions of a corpuscle; and, second, that $\frac{\delta T}{T}$ be greater than $\frac{1}{4} H^2 \frac{e^2}{m^2} T^2$. As to the first condition, our lack of knowledge regarding molecular structure makes it difficult to decide whether it is satisfied or not. But there seems to be no evidence, at any rate, that it is not satisfied. From the second condition we can determine the order of magnitude of the change in the free time which is necessary in order to result in an increase of resistance in a magnetic field. But it is first necessary to get an estimate of the value of T.

The change in the free time of a corpuscle we have supposed due to a change in the configuration of the molecules brought about by the magnetic field. For the ordinary para- and diamagnetic metals, for which the magnetic susceptibility appears to be constant, independent of the strength of the magnetic field, this change in configuration must be supposed to increase with the magnetic field. So that if these metals show an increase in resistance for certain values of the magnetic field we would expect an increase in resistance in all magnetic fields, however great. This is the result obtained by experiment.

With the ferromagnetic metals the case is altogether different. These metals show the phenomenon of magnetic saturation. Up to a certain limit we may suppose that an increase in the magnetic force produces an increased change in the molecular configuration. After this limit has been reached, on further increasing the magnetic force no more change in the molecular configuration is produced, and

consequently, no further change in the free time of a corpuscle. After a certain limit, therefore, the first term on the right of (9) remains constant, while the second term increases with H^2. It appears very probable, then, that for a certain value of H the second term on the right in (9) may become equal to the first term, and for greater values of H become larger than the first term. This would therefore result in a decrease of resistance due to the magnetic field; this is the result obtained by Grunmach experimentally.

For large values of the magnetic force, in the case of iron, for example, we can neglect the first term on the right in (9) in comparison with the second term, and obtain thus:

$$\frac{\delta R}{R} = -\tfrac{1}{4} H^2 \frac{e^2}{m^2} T^2.$$

From Grunmach's results, we have, for iron:

$$H = 2 \times 10^4, -\frac{\delta R}{R} = 10^{-3}, \quad \frac{e}{m} = 10^7,$$

and hence:

$$T = 3 \times 10^{-13}.$$

This gives the order of magnitude of the free time of a corpuscle in iron, and it will probably be not very different in any other metal. We can now find the order of the magnitude of the change in the free time of a corpuscle which is necessary to get an increase of resistance in a magnetic field. For this, we have found the condition to be:

$$\frac{\delta T}{T} > \tfrac{1}{4} H^2 \frac{e^2}{m^2} T^2,$$

taking

$$H = 10^3, \quad \frac{e}{m} = 10^7, \quad T = 3 \times 10^{-13},$$

we get:

$$\frac{\delta T}{T} > 2.5 \times 10^{-6}.$$

It does not seem at all improbable that a magnetic field of the strength assumed should produce a change in T of this order of magnitude.

Having found the order of magnitude of T, we can easily calculate the number of corpuscles per unit volume which are involved in the electric current. We find, by (4)

$$\frac{1}{R} = \frac{1}{2}\frac{e^2}{m} nT.$$

For iron, R is approximately 10^4; $e = 10^{-20}$ in electromagnetic units. Therefore:

$$n = 7 \times 10^{21}.$$

Taking $U = 10^7$ as a probable value of the velocity of the corpuscles inside a metal, we get for the free path:

$$L = 3 \times 10^{-6}.$$

The effect of molecular configuration on the resistance of a conductor is well illustrated in the case of a distinctly crystalline metal like bismuth. In bismuth it is known that the resistance is different in different directions; this may well be explained by the differences in the free paths of corpuscles in different directions.

PRINCETON, N. J.

A HYDRAULIC ANALOGY OF RADIATING BODIES FOR ILLUSTRATING THE LUMINOSITY OF THE WELSBACH MANTLE.

BY R. W. WOOD.

THE device about to be described enables us to illustrate to a class the behavior of different types of radiating bodies when introduced into a flame, and will be found especially useful in explaining the remarkable luminosity of the incandescent mantles used in modern gas-lighting. It is of course not intended to explain the mechanics of radiation, but merely to enable us to describe certain phenomena in terms of easily grasped notions.

Students are told that the more powerfully a body absorbs, the more powerfully will it emit when heated, this relation holding for every individual wave-length. Black bodies then give out the most light when heated. The fact that a white block of lime is far more luminous than a carbon rod when heated in the oxy-hydrogen flame is not usually cited in support of this law, while the fact that the most luminous body of all, the Welsbach mantle, is also quite white is equally unsatisfactory as an illustration, for white bodies are in reality transparent, that is they are made up of masses of small transparent particles, and transparent bodies ought not to emit at all. It is of course necessary to define just what we mean by transparency in this case, and it may be well to consider first a somewhat analogous case. The absorption which is accompanied by high emissivity is true absorption, and not selective reflection,—which is sometimes confused with absorption. A highly reflecting polished metal surface is a poor radiator, but by properly constructing its surface we may give it the power to absorb and emit. A bundle of polished steel needles with their points all turned towards the source of light reflects scarcely any light at all, the rays undergoing multiple reflections between the conical ends of the needles. Such a bundle of needles should emit much more powerfully than a polished steel

surface, and it is easy to see just why it should do so. Each needle, seen end on, sends not only emitted light to the eye, but reflects rays coming from its neighbors. The surface formed by the points of the needles can be regarded as an absorbing surface, which absorbs in virtue of its structure: it is analogous to the hollow "black bodies" with which we are now familiar.

The point which I wish to emphasize is that such a surface, which absorbs not at all in virtue of its molecular nature, is also a powerful radiator, the mechanism by which its radiating power has been increased, being as indicated above. Suppose now we take a perfectly transparent body, which like a perfect reflector has no emitting power. A bead of microcosmic salt (sodium pyro-phosphate) heated in a blast lamp is a good example. Though the platinum wire which supports it glows with vivid incandescence, the bead remains perfectly dark. A glass bead however emits a good deal of light, doubtless from the fact that its transparency is much less at high temperatures, a very common behavior of transparent substances. The microcosmic salt on cooling becomes traversed by hundreds of cleavage planes, which give it a milky appearance. On reheating it it emits light strongly until it finally fuses into a transparent drop, when it instantly becomes dark again. The reason for this behavior is not quite as apparent as in the case of the needles. In fact I am not quite sure that I understand it at all. Quartz behaves in the same way. A drop of clear fused quartz, heated in the blast, emits little or no light, but if it contains spots made up of an emulsion of quartz and air, these spots emit strongly. In other words an opacity resulting from a pulverization of the transparent medium seems to be accompanied with a strong emitting power. Apparently we cannot apply the same reasoning as in the case of the needles, and it looks rather as if the radiation was largely a surface effect. If this is so, it is obvious that an increase of the surface, by enclosures of air, will increase the radiating power. It is my intention to make some measurements of the intensity of the light radiated from the ends of long and short cylinders of red-hot glass.

The hydraulic analogy of radiating bodies which we will now consider occurred to me during a lecture on radiation, and proved quite useful in explaining the different behavior of various types of radiators.

The radiator is represented by a tall hollow cylinder, open at the top and closed at the bottom, provided with a number of outflow pipes of different sizes as shown in Fig. 1. Water flows into the cylinder at a certain definite rate from a horizontal pipe or flume, the height of which (T_2) above the base of the cylinder represents the temperature of the flame. Obviously the level of the water in the cylinder will rise until the rate at which the water flows out exactly equals the rate at which it flows in. This height (T_1) is thetemperature which the radiator acquires in the flame. The jets of water which issue from the tubes represent radiation of different wave-lengths, the small jets representing the short waves. Their velocity corresponds to *intensity* of radiation. We will first suppose our hydraulic radiator to represent a black body, say a lump of carbon. In this case all of the pipes at the bottom are wide open and we have the maximum outflow of all wave-lengths for any given temperature, *i. e.*, for any given height of the fluid within the cylinder. (If we take the cylinder empty, and plunge it into water, jets will squirt into it through the pipes, that is it is a perfect absorber for all wave-lengths.) With all of the pipes open however the level of the water within the cylinder will not rise to any great height, owing to the limited rate at which water flows in from the horizontal pipe. This means that the lump of carbon in the flame does not rise to a very high temperature because it radiates energy at a high rate. At the low temperature there is comparatively little visible light in the radiation, for the shorter waves only appear in quantity at high temperatures. We can imitate this condition in our hydraulic model if we choose by putting valves on the inside of the tubes, those on the small tubes opening only at high pressures.

Fig. 1.

To make our model imitate the bead of microcosmic salt we plug up all of the pipes. The cylinder now represents a transparent body. If immersed in water it absorbs nothing through the pipes, and no matter how high the level of the water rises in it when water is poured in there is no emission of fluid, in other words no radiation.

The body rises in temperature until the temperature is equal to that of the flame, but there is no radiation. Take next the case of the lime in the oxyhydrogen flame. It is a partially transparent substance, and we can imitate it by plugging the tubes with glass beads or cotton. Owing to the lesser rate at which the water now flows out through the tubes, the level rises much higher than when the tubes are all open, and owing to the greater pressure (temperature) we have liquid jets through the small tubes (short wave-length radiation). The inferiority in the emissivity is more than made up for by the higher temperature which the body can acquire. We are now ready for the Welsbach mantle. It has been conclusively shown by Rubens, that the peculiar brilliancy of the thorium mantles caused by a small trace of cerium, is due to the fact that the cerium makes the thorium selectively absorbing for the short waves at high temperatures. If we wave a bunsen flame over a mantle in a brilliantly lighted room, it will be seen to turn yellow at a temperature a little below a red heat. In other words it becomes a strong absorber for the short waves. It is however transparent for the long waves, consequently it does not emit energy at anything like the rate at which a black body does, and in consequence can rise to a high temperature in the flame, exactly as a pure thorium mantle does. Its band of absorption in the blue region enables it to pour out visible radiations nearly as powerfully as those which a black body at the same temperature would emit, hence its enormous brilliancy. Our hydraulic model, with all of its tubes plugged with cotton represents the mantle of pure thoria, while to transform it into the Welsbach mantle we have only to pull out the porous plugs from some of the smaller tubes. In this condition, owing to the impeded flow in the large tubes, the water will rise in the cylinder to a great height, and we get very powerful jets from the small tubes which we have opened, much more powerful than in either of the previous cases considered. Of course with all of the tubes open we could get equally intense small jets if we poured the water in at the top at a sufficient rate. There is a limit to this rate however, for it is obvious that the rate at which the water is poured in at the top, corresponds to the rate at which the flame can pour energy into the radiating body, a circumstance which depends on the conductivity of the body for heat and other things.

It is, of course, not necessary to make the hydraulic apparatus, for its action is so easily understood that a diagram answers every purpose. Its utility lies in the fact that it fixes in the mind of the student the behavior of different types of radiators, when plunged into a flame.

It could be made perhaps to illustrate the displacement of the point of maximum energy in the spectrum, which accompanies a rise in temperature, but it is doubtful whether any such complications would prove beneficial. It seems best on the whole not to try to illustrate too much with it, as its relation to a radiating body is at best rather far-fetched.

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ON THE ESTABLISHMENT OF THE HIGH TEMPERATURE SCALE.[1]

By C. W. Waidner and G. K. Burgess.

IN the interval 400° to 1200° C. the temperature scale is defined by the freezing points of certain pure metals, and in practice is usually reproduced with thermocouples. This scale, which is based primarily on the gas thermometer measurements of Holborn and Day, is the same as that used at the Physikalisch-Technische Reichsanstalt and the National Physical Laboratory.

Reproducibility of Scale. — Experiments undertaken to determine how exactly the scale may be reproduced by the use of metals from various sources, sold as chemically pure, show that Sn, Zn, Cu, and Au may be obtained in this country of sufficient purity to reproduce the above scale to 0°.5 or better. Sb showed differences of 10° or more, and Al about 3°.

Optical Scale. — The scale has been extended above 1200° by optical pyrometers whose calibration is based on the Wien-Planck equation. The realization of this scale necessitated the construction of an experimental black-body in which the temperature distribution could be exactly controlled. The uniformity of temperature attained was about 1° at 1300° throughout two-thirds of the radiating enclosure 30 cm. long.

The high temperature scale was then fixed by the calibration of five pyrometer lamps in terms of the radiation from this nearly ideal black-body whose temperature was measured with thermocouples. By the use of calibrated absorbing glasses and sector discs this scale has been extended to the highest attainable temperatures. The order of agreement of the lamps is well within 2° at 1500°.

Various types of electrically heated radiators were studied to determine the departure of their radiation from ideal black-body radiation.

[1] Communication from the National Bureau of Standards.

Radiation from Platinum and Palladium. — Platinum is useful as a radiating source in many investigations at high temperatures. Further work has been done on the monochromatic radiation from Pt and also from Pd, both in air and hydrogen. The measurement of the temperature of the radiating surface and the changes which the surface undergoes have added greatly to the difficulties of the problem. Three independent methods of temperature measurement were used, one based on the indication of a thermocouple within a platinum spiral, another based on the melting of minute particles of metal on the platinum, and a third on the indications of a thermocouple heated in a colorless gas flame to the same brightness as the radiating platinum. The results are sumarized in the following table, where $t°$ is the true temperature and $s°$ the corresponding black-body temperature for red ($\lambda = 0.66\mu$) and green ($\lambda = 0.55\mu$), as determined in air.

$s° =$	700°	800°	900°	1000°	1100°	1200°	1300°	(1400°)	(1500°)
(red) $t°-s° =$	46°	58°	72°	90°	110°	132°	155°	(178°)	(200°)
(green) $t°-s° =$	—	—	53°	68°	86°	103°	121°	139°	(155°)

The difficulty of determining the temperature of a radiating surface limits the accuracy of these results to about 10°.

Platinum and palladium agree in their radiating properties to within the limits of precision attainable to 1250°.

Determination of Fixed Points. — For the temperature scale above 1200° C. it is of great importance to determine accurately a number of fixed points. The melting points of palladium and platinum, which are especially suitable for this purpose, have been made the subject of numerous researches, but are still uncertain by some 40°.

Determinations of these constants were made by observing with an optical pyrometer the temperatures of melting within an iridium tube furnace heated with an alternating current. The temperatures, as given in the following table, are based on the extrapolation of Wien's equation, $J = c_1\lambda^{-5}e^{-\frac{c_2}{\lambda T}}$, assuming $c_2 = 14500$.

Melting Points of Pd and Pt in Iridium Furnace.

Metal.	Wave Length.	Absorption Coefficient.	Number of Observations.	Melting Point.	Average Deviation of an Observation.
Pd	0.666 μ	Glasses, $k=200$	8	1546°C	2°.5
	.666	$k=1$	7	1553°	2°.0
	.666	Glasses, $k=200$	7	1545°	2°.5
Pt	.668	Sector Disc $k=35.4$	10	1753°	2°.0
	.668	Glasses, $k=199$	16	1754°	3°.5
	.668	Glasses, $k=199$	7	1752°	5°.0
	.547	Sector Disc $k=35.4$	6	1748°	2°.0
	.462	Sector Disc $k=35.4$	4	1749°	3°.0

At least three pyrometer lamps were used in each series. The second value found for *Pd* is an independent extrapolation of the calibration equations of the lamps and therefore not based on Wien's equation.

Simultaneous measurements were made with thermocouples of the following types: Pt, 90 Pt–10 Rh; Pt, 90 Pt–10 Ir; 90 Pt–10 Rh, 80 Pt–20 Rh; Ir, 90 Ir–10 Ru. The results obtained differ by as much as 40° at the Pt point for the same couple according to the extrapolation formulæ used. The usual formulæ

$$(a) \qquad E = -a + bt + ct^2$$

$$(b) \qquad E = mt^n$$

gave, for the 10 per cent. Rh and Ir couples, the following values when calibrated at the freezing points of the metals indicated. Con-

Melting Points with Thermocouples.

Metal.	Equation (a).	Equation (b).	
	Zn, Sb, Cu.	Zn, Cu.	Sb, Cu.
Palladium.	1530°	1544°	1542°
Platinum.	1706	1732	1730

vincing evidence is at hand that formula (a) gives low values of temperature when extrapolated beyond 1200° C.

The melting points of Pd and Pt were also determined by observing the black-body temperatures of ribbons of these metals at the instant of melting.

Melting Points Based on Platinum Radiation.

Metal.	Wave Length.	S° by Wien's Equation.	S° by Lamp Extrapolation.	Melting Point.
Pd	.652 μ	1389°		1563°
	.662		1386°	
	.547		1425	
	.462		1450	
Pt	.666	1534		1750
	.666		1544	
	.547	1578		
	.547		1593	
	.462		1627	

The melting point of Pd is higher by this method because the assumption is made that Pd and Pt have the same emissivity. Observations indicate that the values of $t°-s°$ increase less rapidly at high temperatures for Pd.

The present investigation[1] gives for the melting points of palladium and platinum, **1546°** and **1753°**, respectively, based on the extrapolations of the Wein-Planck equation, in which the constant c_2 is taken as 14500, and using as the melting point of gold, 1064°. If c_2 were taken as 14200,[2] these melting points would become about 1555° and 1760°. On account of the more satisfactory theoretical basis of the Stefan-Boltzmann law, which also gives extrapolated temperatures on the thermodynamic scale, it is planned to attack the problem from this standpoint.

WASHINGTON, D. C.,
February 19, 1907.

[1] A more complete account of this investigation is given in Bulletin, Bureau of Standards, vol. 3, 1907.

[2] Holborn and Valentiner, Ann. d. Physik, **22**, p. 1, 1907.

THE VAPOR PRESSURE OF SOLID CARBONIC ACID.

By H. du Bois.

IN 1899 I published with A. P. Wills some preliminary measurements relative to the thermometric and cryogenic applications of solid carbonic acid.[1] The object was to establish some approximate data with reference to practical applications of solid carbonic acid in the laboratory, and in no wise to attempt an exact final determination of the temperature-pressure function. On account of the return of Professor Wills to America, the proposed more accurate determinations referred to in the last part of the original article were not made.

Recently there have appeared two articles relative to this subject, one by J. Zeleny and R. H. Smith, and one by J. and A. Zeleny.[2] The $P\theta$-curve found lies about one degree higher than ours, and this is displaced toward the axis of pressure; instead of the value 5.5 cm. Hg. per degree for $dP/d\theta$ at atmospheric pressure found by us, the first of the above mentioned articles gives the value 6.35, and the latter the value 5.87, *i. e.*, 1 cm. Hg. corresponds to a temperature change of .17°.

In accordance with the above introductory remarks it is quite possible that such a difference might exist; also it is well known that the graphical determination of the slope of a curve with only a few given points is much more uncertain than the inverse graphical integration.

In a final determination of the constants it seems to me that the employment of pure carbonic acid is absolutely necessary, but the physicists mentioned above used the material as it is to be bought, as did likewise we ourselves. Our material was obtained through the burning of coke, and 99.5 per cent. at least was carbonic acid although there might have been traces of air, water and grease.

In America, as I am informed by reliable authorities, liquid carbonic acid is very frequently obtained from brewery gases and is not in general so pure as that obtained here, since it frequently unavoidably contains odoriferous material and the like, associated with the brewing process. It is scarcely possible to judge without further information whether, or in how far, such circumstances are adequate to account for the differences referred to above.

Berlin, November 5, 1906.

[1] H. du Bois und A. P. Wills, Verh. D. Phys. Ges., I., 168, 1899.

[2] J. Zeleny and R. H. Smith, Phys. Zeitschr., 7, 667, 1906; J. und A. Zeleny, l. c., p. 716; Phys. Rev., 23, 308, 1906.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

NOTE ON THE HISSING METALLIC ARC.[1]

BY W. G. CADY.

THE arc between iron electrodes in pure nitrogen was investigated in order to determine whether the presence of oxygen plays the same part in causing the hissing stage of the iron arc as is the case with the carbon arc. It was found that the iron arc in nitrogen resembles the hissing rather than the quiet arc. No quiet state, properly speaking, could be obtained, as the arc went out whenever the current was decreased to the point where the positive electrode ceased to be vaporized. This suggests that the quiet state of the metallic arc may be defined as that in which the material of the anode is not heated to the boiling point. All observations on the iron arc in air, including rate of consumption of electrodes, temperature of oxid globules as determined by a thermo-element, and appearance of the spectrum of the arc between iron and copper, tend to confirm this view.

THE TEMPERATURE OF THE MERCURY ARC.[2]

BY CHAS. T. KNIPP.

THE purpose of this investigation was to determine the temperature of the mercury arc as recorded by Cu-Pt thermo-couples. The form of the tube was that used in the Hewitt lamp. The electrodes were replaced by mercury columns supported by atmospheric pressure. A third electrode (mercury column) was inserted midway between the terminals of the lamp. The tube was exhausted by a power Geryk pump which was kept running continuously while taking the observations. The temperature measurements were made by inserting the couples, each protected by a thin walled glass tube, up through the mercury columns and adjusting them to any desired height. The end couples were placed 2

[1] This paper, presented at the meeting of March 2, 1907, is supplementary to one on 'The Hissing Point of the Metallic Arc" read at the meeting of the Physical Society, in New York, December 29, 1906. See PHYS. REV., p. 381, April, 1907.

[2] Abstract of a paper presented at a meeting of the Physical Society held December 27-29, 1906.

cm. above the surfaces of the mercury. It was found that the galvanometer deflections increased steadily as the distance above the surface of the mercury increased. The form of the tube prevented making this distance greater than 2 cm. as it was desired to keep the three junctions on the axis of the tube. The length of the tube was 20 cm., and its diameter 3.8 cm.

Adjusting the current to 4 amperes — which value caused the lamp to give out approximately the same illumination as in the commercial lamp — and a potential difference of 20.7 volts, the temperature as recorded by the junctions was 147° C. at the anode, 164° C. at the cathode, and 178° C. at the point midway between the anode and cathode. In general, for values of current ranging from 3 to 9 ampers, the cathode was found to be hotter than the anode, while the midway point was hotter than either. Again, the individual readings obtained at the cathode fluctuated much more than those obtained at the anode, while there was scarcely any variation from a smooth curve either for increasing or decreasing currents at the midway point.

On the Current Carried by Canal Rays in a Discharge Tube.[1]

By C. C. Perry.

THE method employed in this work was that of allowing canal rays to penetrate into an earthed Faraday cylinder, one end of which served as kathode in the discharge tube. The rays were allowed to fall on an insulated brass electrode, which was connected to earth through a D'Arsonval galvanometer. The deflections of the galvanometer could then be used as a measure of that part of the total discharge current which was carried into the Faraday cylinder by the canal rays. The galvanometer circuit was so arranged by means of suitable mercury cup switches, as to first allow the "canal ray current" to be measured, and then the total current carried by the discharge. The ratio of the two, canal ray current to total current, was taken (for different gas pressures) as a measure of the relative numbers of canal ray particles present in discharges at these pressures, provided the discharge potential be kept constant.

Several such series of measurements were made with gas fillings of air, oxygen and hydrogen. When these measurements are plotted, with the ratios of canal ray current to total current as ordinates, and the gas pressures as abscissae, the curves in every instance exhibit a maximum ordinate for pressures which in every case observed were in excess of the

[1]Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1907.

pressures at which canal rays show their maximum fluorescent effects. (.035 to .04 mm. of Hg.). This pressure for the maximum production of canal rays as measured by the galvanometer, occurs always at pressures not far removed from those at which the first kathode ray fluorescence becomes visible, and appears to be different for different gas fillings.

Several sources of error were investigated. Among these, the most important seemed to be, 1st, as to whether phenomena could be attributed to the diffusion of ions from in front of the kathode, and, 2d, as to whether the phenomenon of the production of secondary rays (soft kathode rays) as described by J. J. Thomson [1] and Austin [2] could account for the maximum in the curves.

The first of these was shown to be improbable by the introduction of a sufficient back e.m.f. in the circuit from the insulated electrode to earth, to preclude any diffusion phenomena. The second was tested by applying a magnetic field parallel to the surface of the insulated electrode, and of sufficient strength to turn completely back any secondary rays while not strong enough to disturb the main part of the discharge. In this way the observations of Thomson and Austin were confirmed, the effect observed was a diminution of the current from the electrode to earth when the field was on. This did not however change the general form of the curve, for if two sets of readings were taken, at the various gas pressures, one with the field on and one with it off, the corrected curve still showed a maximum slightly displaced from its former position. These secondary rays appear to begin at a perfectly definite gas pressure and hence a perfectly definite kinetic energy for canal rays in each gas tried, if the discharge potential is kept constant. The magnitude of the secondary effect increases with increasing mean free path.

The following conclusions have been deduced: 1°. Canal rays are present in a discharge tube in the greatest numbers at gas pressures so great that they do not acquire energy enough to be detected by fluorescence effects.

2°. The velocity at which this maximum occurs, appears to be connected in some way with the beginning of kathode rays in the discharge, and also to vary with the gas filling.

Further work is in progress to test if possible, the increase of kathode rays as the canal rays disappear, and also to measure the velocity of the canal rays at their point of maximum number.

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[1] J. J. Thomson, Proc. Camb. Phil. Soc., Vol. 13, part 4, p. 212, 1906.
[2] Austin, PHYS. REV., Vol. 22, pp. 312–319, 1906.